Analysis of semiclassical approximations in the framework of Quantumhadrodynamics

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Abstract

We examine the importance of gradient corrections to the Thomas-Fermi (TF) approximation for the description of semi-infinite nuclear matter and nuclei within the framework of Quantumhadrodynamics. The discussion of semi-infinite nuclear matter exhibits most clearly how surface properties depend on the parametrisation of the effective interaction. In agreement with earlier studies it is found that for parametrisations with low effective mass the original extended TF (ETF) equations can not be solved. We present a scheme to overcome this problem which is very much in the spirit of the semiclassical approach and allows for a solution in any situation. For the case of nuclei the results of the TF and ETF approximations are compared to those of Hartree calculations which serve as a standard in this context. In accordance with the sensitivity of semi-infinite nuclear matter results to the specific parametrisation, ETF does not yield a consistent improvement over TF for all parametrisations. However, by examining a larger number of parametrisations it is shown that those cases where TF results are closer to Hartree data have to be regarded as fortuitous and that on average the agreement between ETF and Hartree results is clearly better.

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1 Introduction

The concept of a phenomenological effective nucleon-nucleon interaction understood as a parametrisation of the density dependent effective G-matrix, has been used extensively for the description of nuclear properties [1]. Among the various phenomenological interactions the Skyrme force [2] has gained much attraction due to its ability to reproduce binding energies and nuclear radii over the entire periodic table within the Hartree-Fock (HF) approximation [3]

As an alternative to microscopical HF calculations it has been attempted to treat the nuclear many body problem in the density functional formalism (sometimes called energy density formalism in the context of nuclear physics). Based on the Hohenberg-Kohn theorem [4] which emphasizes the prime importance of the ground state density ρ the Skyrme HF energy $E_{HF}[\rho, \tau, \underline{J}]$ is turned into a density functional by expressing the kinetic energy density τ and the spin orbit density $\underline{J}$ as a functional of ρ and its derivatives [5]. The ground state density is then determined by variation. The most elaborate applications of this method include inhomogenity corrections up to fourth order in the functionals $\tau[\rho]$ and $\underline{J}[\rho]$ as well as contributions from the effective nucleon mass. As a general result it has been found [6] that second order calculations always lead to an overbinding of finite nuclei and underestimate their radii. Only the more sophisticated fourth order calculations are able to reproduce average nuclear ground state properties sufficiently well [7, 8, 9] when they are compared with Strutinsky averaged HF results.

The phenomenological Skyrme interaction relies upon the assumption of a static two nucleon potential and does not take into account either relativistic effects or mesonic degrees of freedom. Even for nuclear structure calculations where the rather small binding energy seems to confirm the nonrelativistic approximation it has been claimed that relativistic effects may be important as the small binding energy can result from a cancellation of large potential energy contributions of opposite sign. Without doubt, relativistic effects become essential when one intends applications under extreme con-

ditions, e.g. neutron stars or heavy ion collisions, where the nuclear density is much larger than the nuclear matter value.

Based on initial work of Duerr [10], a relativistic field theoretical description of the nucleon-nucleon interaction which accounts for mesonic degrees of freedom has been developed by Walecka and collaborators [11, 12]. In the simplest version the nucleons interact by the exchange of an attractive scalar σ-meson and a repulsive vector ω-meson. Rather than deriving a bare nucleon-nucleon interaction by fitting the coupling constants and masses of the mesons to nucleon-nucleon scattering data, these parameters are fixed by nuclear matter properties and some additional requirements. In this way a phenomenological relativistic effective interaction is derived which incorporates various additional effects, e.g. the exchange of further mesons, multiple meson exchange etc. However, the parameters of the interaction are as well dependent on the many body approximation. In most applications the effective interaction is used in the mean field (MF) approximation, neglecting exchange and correlation contributions as well as vacuum polarisation effects (no sea approximation).

The original σ-ω model has been extended in several respects. Boguta and Bodmer [13] introduced a selfinteraction of the scalar meson leading to a density dependence of the effective interaction which is necessary for an adequate description of nuclear properties. The electromagnetic interaction between the protons requires photon exchange and a ρ vector meson is added in order to account for the neutron excess in heavy nuclei. Extensive calculations performed in the mean field approximation [14, 15] demonstrate that the quality of the relativistic approach is comparable to nonrelativistic HF calculations with the Skyrme force. However, the relativistic approach has the advantage that the spin orbit force emerges quite naturally from the relativistic formulation.

Encouraged by the success of the density functional formalism in the framework of nonrelativistic nuclear physics it seems desirable to investigate the potential of this formalism in the relativistic case. In a recent paper [16] we have given the foundations for an application of density functional theory to QHD by generalizing the

Hohenberg-Kohn theorem. The first calculations of finite nuclei in the context of the density functional formalism relied upon the TF approximation [17, 18, 19], neglecting inhomogenity corrections altogether. Recently, corrections of order $\hbar^2$ were derived by Centelles et al. [20] as well as in Ref.[16] at the mean field level of QHD, whereas a similar investigation by Weigel et al. [21] starts from the HF approximation taking into account Fock contributions to the mean fields. First results for finite nuclei in the extended TF (ETF) approximation were reported in recent publications by Centelles et al. [22, 23] and Von-Eiff and Weigel [24]. One conclusion of [22] is the strong dependence of the TF and ETF approximations on the effective interaction chosen, which makes it rather difficult to judge the importance of inhomogenity corrections.

The aim of the present investigation is to examine in more detail the dependence of the TF and ETF approximations on the effective interaction. We try to identify those parameters of the interaction which mainly influence the properties of infinite nuclear matter, semi-infinite nuclear matter and finite nuclei with the purpose to draw definite conclusions about the quality of each density functional approximation.

This paper is organized as follows. The Lagrangian and the Hamiltonian of the extended σ-ω model are recalled in Section 2 followed by a short discussion how to derive the basic functional of the (effective) kinetic energy for this more general case. Subsequently the variational equations are derived and the boundary conditions are discussed under which these equations were solved. Section 3 is concerned with the semi-infinite nuclear matter problem in order to investigate the behaviour of the nuclear surface in the TF and ETF approximations, with special attention paid to the dependence on the effective interaction. To overcome difficulties with the original scheme of solving the ETF variational equations we propose in Section 4 an alternative scheme and discuss its relation to the original one. Turning to the case of finite nuclei in Section 5 we compare results of TF, ETF and Hartree approximations in order to draw conclusions about the importance of the inhomogenity corrections. Finally we summarize our results in Section 6. In Appendix A we collect some basic relations for infinite nuclear

matter which are used to relate nuclear matter properties to the parameters of the effective interaction.

Throughout the paper we use the units $\hbar = c = 1$.

2 The ETF approximation

Since the pioneering work of Walecka et al.[11, 12], in which the basic principles of a quantum field theoretical description of nuclear systems were introduced, the original σ-ω-model has experienced a number of refinements. Presently the standard version of QHD [14, 15] describes the nuclear interaction via the exchange of several bosons: The scalar σ-meson responsible for the nuclear attraction and the vector ω-meson representing the short range repulsion are supplemented by the photon and the isovector ρ-meson taking into account the asymmetry between protons and neutrons. Furthermore, as first proposed by Boguta and Bodmer [13] a selfinteraction of the scalar meson is introduced resulting in the Lagrangian

$$\begin{aligned}
\mathcal{L} &= \bar{\Psi}\left\{ i\gamma^\mu\partial_\mu - (M - g_s\phi) - \gamma^\mu(g_v V_\mu + \frac{e}{2}(1+\tau_3)A_\mu + \frac{g_\rho}{2}\tau\cdot\mathbf{b}_\mu)\right\}\Psi \\
&\quad + \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m_s^2\phi^2) - \frac{1}{3}b\phi^3 - \frac{1}{4}c\phi^4 \\
&\quad - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_v^2 V_\mu V^\mu - \frac{1}{4}H_{\mu\nu}H^{\mu\nu} \\
&\quad - \frac{1}{4}\mathbf{B}_{\mu\nu}\cdot\mathbf{B}^{\mu\nu} + \frac{1}{2}m_\rho^2\mathbf{b}_\mu\cdot\mathbf{b}^\mu
\end{aligned} \tag{1}$$

where

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu \tag{2}$$

$$H_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{3}$$

$$\mathbf{B}_{\mu\nu} = \partial_\mu\mathbf{b}_\nu - \partial_\nu\mathbf{b}_\mu - g_\rho(\mathbf{b}_\mu\times\mathbf{b}_\nu). \tag{4}$$

Here boldface letters denote vectors in isospin space.

In the mean field approximation the quantum fields ϕ, V_μ, A_μ and $\mathbf{b}_\mu$ are replaced by classical time-independent potentials. The additional assumptions of time reversal

symmetry and charge conservation lead to further simplifications [15], i.e. only the timelike components V_0,A_0 and $b_0 \equiv b_{00}$ of the meson fields survive. Under these restrictions the energy may be represented as

$$\begin{aligned} E = \int d^3x \Big\{ \quad & < g|\Psi_p^\dagger(-i\underline{\alpha}\,\underline{\nabla} + \beta(M - g_s\phi) + V_p)\Psi_p|g > \\ & + < g|\Psi_n^\dagger(-i\underline{\alpha}\,\underline{\nabla} + \beta(M - g_s\phi) + V_n)\Psi_n|g > \\ & + \frac{1}{2}[(\nabla\phi)^2 + m_s^2\phi^2] + \frac{1}{3}b\phi^3 + \frac{1}{4}c\phi^4 \\ & - \frac{1}{2}[(\nabla V_0)^2 + m_v^2 V_0^2] - \frac{1}{2}(\nabla A_0)^2 \\ & - \frac{1}{2}[(\nabla b_0)^2 + m_\rho^2 b_0^2] \Big\}, \end{aligned} \tag{5}$$

where we have decomposed the nucleon field operator into its isospin components in order to illustrate that protons and neutrons move in different potentials,

$$V_p = g_v V_0 + \frac{g_\rho}{2} b_0 + eA_0 \tag{6}$$

$$V_n = g_v V_0 - \frac{g_\rho}{2} b_0, \tag{7}$$

and V_0, ϕ, b_0 and A_0 are now understood as mean field potentials. Consequently one has to distinguish between the proton and neutron densities ρ_p and ρ_n, respectively, (or, alternatively ρ_p and $\rho_3 \equiv \rho_p - \rho_n$) as two independent variables.

Our aim is to represent the kinetic energy,

$$T_s = \int d^3x t_s(x) = \sum_{q=p,n} \int d^3x < g_s|\Psi_q^\dagger(-i\underline{\alpha}\,\underline{\nabla} + \beta M)\Psi_q|g_s >, \tag{8}$$

(where $|g_s >$ represents the ground state of that noninteracting system which leads to the same ground state densities as the actual interacting system [16]) as a functional of the densities ρ_p, ρ_n and the scalar densities ρ_{sp} and ρ_{sn}. Since T_s is the kinetic energy of an equivalent noninteracting system the proton and neutron contributions seperate according to

$$T_s = T_s[\rho_p, \rho_{sp}] + T_s[\rho_n, \rho_{sn}]. \tag{9}$$

The derivation of an approximate $T_s[\rho, \rho_s]$ for the σ-ω model is explicitly given in [16]. After a semiclassical expansion of the density, the scalar density and the kinetic energy

density in terms of auxilliary potentials to second order in $\hbar$ (or equivalently gradients of the potentials), the kinetic energy is recast as a density functional by inversion, i.e. by eliminating the auxilliary potentials in favour of the densities. This functional is universal in the sense that it holds for any set of local potentials (V,ϕ) and thus in particular for the present situation.

The elimination of the scalar potential through the scalar density, however, is not compelling. Following Centelles et al.[20] one might alternatively represent the kinetic energy as a functional of ρ_p, ρ_n and the scalar mean field potential ϕ,

$$T_s = \tilde{T}_s[\rho_p, M^*] + \tilde{T}_s[\rho_n, M^*] = T_{sp} + T_{sn}, \tag{10}$$

where the effective mass is defined in terms of the scalar mean field potential as $M^* = M - g_s\phi$. As the scalar densities ρ_{sp} and ρ_{sn} couple to the same scalar potential there is only one effective mass entering eq.(10). In addition, the scalar density itself is no longer understood as a fundamental variable but rather written as a functional of ρ_p, ρ_n and M^*,

$$\rho_s = \rho_s[\rho_p, M^*] + \rho_s[\rho_n, M^*] = \rho_{sp} + \rho_{sn}, \tag{11}$$

where ϕ now takes the role of an independent quantity.

From a numerical point of view it turns out to be more useful to choose the set ρ_p, ρ_n and M^* as independent variables. Replacing the kinetic energy contribution in eq.(5) by $T_s[\rho_p, \rho_n, M^*]$ defines the energy in the ETF approximation

$$\begin{aligned} E = \int d^3x \Big\{ \quad & t_s - g_s\phi\rho_s + g_vV_0(\rho_p + \rho_n) + eA_0\rho_p + \frac{1}{2}g_\rho b_0(\rho_p - \rho_n) \\ & + \frac{1}{2}[(\nabla\phi)^2 + m_s^2\phi^2] + \frac{1}{3}b\phi^3 + \frac{1}{4}c\phi^4 \\ & - \frac{1}{2}[(\nabla V_0)^2 + m_v^2V_0^2] - \frac{1}{2}(\nabla A_0)^2 \\ & - \frac{1}{2}[(\nabla b_0)^2 + m_\rho^2 b_0^2] \Big\}. \end{aligned} \tag{12}$$

For the derivation of variational equations and the evaluation of energies it is useful to define an effective kinetic energy density (compare [20]),

$$t_s^* = \sum_{q=p,n} t_{sq}^* = \sum_{q=p,n} (t_{sq} - g_s\phi\rho_{sq}), \tag{13}$$

which to second order is given by

$$\begin{aligned} t^*_{sq} &= \frac{1}{8\pi^2}(k_q^3\epsilon_q + k_q\epsilon_q^3 - M^{*4}\ln\frac{k_q+\epsilon_q}{M^*}) \\ &+\frac{1}{24\pi^2}\frac{k_q}{\epsilon_q}\left(1+2\frac{k_q}{\epsilon_q}\ln\frac{k_q+\epsilon_q}{M^*}\right)(\nabla k_q)^2 \\ &+\frac{1}{24\pi^2}\left(\frac{k_q}{\epsilon_q}-(1+2\frac{k_q^2}{\epsilon_q^2})\ln\frac{k_q+\epsilon_q}{M^*}\right)(\nabla M^*)^2 \\ &+\frac{1}{6\pi^2}\frac{k_qM^*}{\epsilon_q^2}\ln\frac{k_q+\epsilon_q}{M^*}(\nabla k_q)(\nabla M^*) \end{aligned} \tag{14}$$

where

$$k_q = (3\pi^2\rho_q)^{\frac{1}{3}}$$

and

$$\epsilon_q = \sqrt{k_q^2 + M^{*2}}.$$

The variational equations are derived by minimizing the total energy with respect to the mean field potentials and the particle densities under the condition of fixed proton and neutron numbers

$$\frac{k_q}{\epsilon_q}\rho_q^{[2]} = \frac{\partial\rho_q}{\partial k_q}(\epsilon_q + V_q - \mu_q) \qquad q = p,n \tag{15}$$

$$(\Delta - m_v^2)V_0 = -g_v(\rho_p+\rho_n) \tag{16}$$

$$(\Delta - m_s^2)\phi = -g_s\sum_{q=p,n}(\rho_{sq}^{[0]}+\rho_{sq}^{[2]}) + b\phi^2 + c\phi^3 \tag{17}$$

$$\Delta A_0 = -e\rho_p \tag{18}$$

$$(\Delta - m_\rho^2)b_0 = -\frac{g_\rho}{2}(\rho_p - \rho_n) \tag{19}$$

where the lowest order contribution to the scalar density is

$$\rho_{sq}^{[0]} = \frac{M^*}{2\pi^2}(k_q\epsilon_q - M^{*2}\ln\frac{k_q+\epsilon_q}{M^*}) \tag{20}$$

and the second order contributions are

$$\begin{aligned} \rho_q^{[2]} &= \frac{1}{12\pi^2}\left(1+2\frac{k_q}{\epsilon_q}\ln\frac{k_q+\epsilon_q}{M^*}\right)\Delta k_q \\ &+\frac{1}{24\pi^2}\frac{1}{k_q}\left(1+\frac{k_q^2}{\epsilon_q^2}+4\frac{k_qM^{*2}}{\epsilon_q^3}\ln\frac{k_q+\epsilon_q}{M^*}\right)(\nabla k_q)^2 \end{aligned} \tag{21}$$

$$
\begin{aligned}
&+\frac{1}{6\pi^2}\frac{M^*}{\epsilon_q}\ln\frac{k_q+\epsilon_q}{M^*}\Delta M^* \\
&-\frac{1}{24\pi^2}\frac{k_q}{\epsilon_q^2}\left(1-4\frac{k_q}{\epsilon_q}\ln\frac{k_q+\epsilon_q}{M^*}\right)(\nabla M^*)^2 \\
&-\frac{1}{12\pi^2}\frac{1}{M^*}\left(1+\frac{k_q^2}{\epsilon_q^2}+4\frac{k_q M^{*2}}{\epsilon_q^3}\ln\frac{k_q+\epsilon_q}{M^*}\right)(\nabla k_q)(\nabla M^*) \\
\rho_{sq}^{[2]} \;=\; &-\frac{1}{6\pi^2}\frac{k_q M^*}{\epsilon_q^2}\ln\frac{k_q+\epsilon_q}{M^*}\Delta k_q \qquad (22)\\
&-\frac{1}{24\pi^2}\frac{1}{M^*}\left(5\frac{k_q}{\epsilon_q}-3\frac{k_q^3}{\epsilon_q^3}+4\frac{M^{*4}}{\epsilon_q^4}\ln\frac{k_q+\epsilon_q}{M^*}\right)(\nabla k_q)^2 \\
&-\frac{1}{12\pi^2}\left(\frac{k_q}{\epsilon_q}-(1+2\frac{k_q^2}{\epsilon_q^2})\ln\frac{k_q+\epsilon_q}{M^*}\right)\Delta M^* \\
&-\frac{1}{24\pi^2}\frac{k_q^2}{\epsilon_q^2 M^*}\left(3\frac{k_q}{\epsilon_q}+4\frac{M^{*2}}{\epsilon_q^2}\ln\frac{k_q+\epsilon_q}{M^*}\right)(\nabla M^*)^2 \\
&+\frac{1}{12\pi^2}\frac{k_q}{\epsilon_q^2}\left(3\frac{k_q}{\epsilon_q}+4\frac{M^{*2}}{\epsilon_q^2}\ln\frac{k_q+\epsilon_q}{M^*}\right)(\nabla k_q)(\nabla M^*).
\end{aligned}
$$

This coupled system of nonlinear differential equations has to be reduced to the symmetry present in the system under consideration. For the case of finite nuclei with spherical symmetry it is supplemented by (following [25])

$$\frac{d}{dr}\mu_q = 0 \qquad q=p,n \qquad (23)$$

$$\frac{d}{dr}N_q = 4\pi r^2\rho_q \qquad q=p,n \qquad (24)$$

where the first two equations determine the unknown chemical potentials μ_q and the second set of equations allow to fix the particle numbers by appropriate boundary conditions for the norm functions $N_q(r)$. Thus one is left with ten coupled nonlinear differential equations, four of first and six of second order. This boundary value problem has been solved using the program package COLSYS [26] which has been successfully applied to similar problems in recent applications [25, 27].

In order to specify the set of boundary values under which these differential equations have to be solved one has to examine the limiting behaviour of the various functions for $r \to 0$ and $r \to \infty$. A power series expansion around the origin shows that

one has to require

$$k_p'(0) = k_n'(0) = V_0'(0) = \phi'(0) = A_0'(0) = b_0'(0) = 0 \tag{25}$$

supplemented by the obvious conditions

$$N_p(0) = N_n(0) = 0. \tag{26}$$

The dominant contributions in the large-r limit are

$$k_p(r) \underset{r\to\infty}{\longrightarrow} a_p \frac{e^{-\gamma_p r}}{r^{\frac{2}{3}+\frac{4M}{\gamma_p}\alpha Z}} \tag{27}$$

$$k_n(r) \underset{r\to\infty}{\longrightarrow} a_n \frac{e^{-\gamma_n r}}{r^{\frac{2}{3}}} \tag{28}$$

$$g_v V_0(r) \underset{r\to\infty}{\longrightarrow} b\frac{e^{-m_v r}}{r} \tag{29}$$

$$g_s \phi(r) \underset{r\to\infty}{\longrightarrow} c\frac{e^{-m_s r}}{r} \tag{30}$$

$$eA_0(r) \underset{r\to\infty}{\longrightarrow} \frac{\alpha Z}{r} - \frac{4\pi\alpha}{9\gamma_p^2}\rho_p(r) \tag{31}$$

$$g_\rho b_0(r) \underset{r\to\infty}{\longrightarrow} d\frac{e^{-m_\rho r}}{r} \tag{32}$$

$$N_p(r) \underset{r\to\infty}{\longrightarrow} Z - \frac{4\pi}{3\gamma_p} r^2 \rho_p(r) \tag{33}$$

$$N_n(r) \underset{r\to\infty}{\longrightarrow} N - \frac{4\pi}{3\gamma_n} r^2 \rho_n(r) \quad , \tag{34}$$

with

$$\gamma_q = \sqrt{8M^2(1 - \frac{\mu_q}{M})}. \tag{35}$$

The exponential decay of the densities is governed by the chemical potentials (note that μ_q still contains the nucleon's rest mass). The same exponential behaviour is found for the nonrelativistic semiclassical approximation to order $\hbar^2$ (Note that in the presence of the electromagnetic interaction ($\alpha = \frac{e^2}{4\pi}$) $k_p(r)$ and $k_n(r)$ show a different asymptotic behaviour). As all solutions apart from the Coulomb potential show an exponential decay one requires the following relations at the right boundary r_{max} of our interval of numerical solution,

$$k_p(r_{max}) \quad = \quad k_n(r_{max}) = V_0(r_{max}) = \phi(r_{max}) = b_0(r_{max}) = 0 \tag{36}$$

$$eA_0(r_{max}) = \frac{Z\alpha}{r_{max}} \quad (37)$$

$$N_p(r_{max}) = Z \quad (38)$$

$$N_n(r_{max}) = N. \quad (39)$$

r_{max} must be chosen such that the neglect of all exponential correction terms in (27-34) is justified within the numerical accuracy required.

3 Semi-infinite nuclear matter

Before discussing the nonlinear σ-ω model a comparison between the TF and ETF approximations is made for the linear model. Though for a realistic discussion of physical properties the linear model has to be extended, the effects of inhomogenity corrections can be extracted more easily in this case. These corrections are expected to show up most directly in surface properties which are usually discussed in the context of semi-infinite nuclear matter.

3.1 Linear σ-ω model

In the linear σ-ω model the saturation properties of nuclear matter

$$\frac{\epsilon(\rho_0)}{\rho_0} = M + a_V \quad (40)$$

$$P(\rho_0) = 0 \quad (41)$$

where M is the nucleon mass, a_V is the binding energy per particle of infinite nuclear matter and ρ_0 its saturation density, do not only determine the dimensionless coupling constants $C_s^2 = M^2 g_s^2/m_s^2$ and $C_v^2 = M^2 g_v^2/m_v^2$ but also the incompressibility parameter K and the effective mass at saturation M_0^*. For an inhomogenous system the meson masses m_s and m_v have to be specified. In most applications (see e.g.[28, 29]) one identifies the vector meson mass with the physical mass of the ω-meson, $m_v = 783 MeV$, and we will follow this choice. Therefore, only the scalar meson mass m_s remains as an adjustable parameter which determines the surface properties of semi-infinite nuclear matter and finite nuclei.

Choosing the saturation properties of nuclear matter according to $k_F^0 = 1.42 fm^{-1}$ and $a_V = -15.75 MeV$ leads to the parameter set

$$C_s^2 = 267.568 \quad C_v^2 = 196.300 \quad K = 545.6 MeV \quad M_0^*/M = 0.5558 \quad .$$

Fig. 1 illustrates the dependence of the surface thickness t (defined to be the region where the density drops from 90% to 10% of its central value ρ_0) on m_s for $400 MeV < m_s < 500 MeV$. With increasing m_s the range of the scalar interaction is reduced, leading to a strong decrease of t for the case of both TF and ETF. The ETF approach always leads to a smaller t than the TF approximation as the inhomogenity corrections tend to compress the surface region. The difference $t(TF) - t(ETF)$ depends only moderately on m_s. It is a slightly increasing function of m_s, i.e. the importance of gradient corrections grows the steeper the surface becomes with m_s, just as one would expect.

In fig. 2 we show the surface energy a_S,

$$a_S = 4\pi r_0^2 \int_{-\infty}^{\infty} dz \, \{\epsilon(z) - (M + a_V)\rho(z)\} \quad , \tag{42}$$

(r_0 is the nuclear matter radius, $r_0 = [3/4\pi\rho_0]^{\frac{1}{3}}$) as a function of m_s for the two approaches. In both approximations the surface energy is decreasing with increasing m_s as the surface regime becomes more and more compressed. The difference $a_S(TF) - a_S(ETF)$ is always positive with only a weak dependence on m_s. We note that the ratio a_S/t is increased by the inclusion of inhomogenity corrections (TF: $a_S/t = 13.26 MeV/fm$, ETF: $a_S/t = 14.49 MeV/fm$, for $m_s = 450 MeV$). As discussed by several authors [13, 30] this ratio is mainly determined by the incompressibility parameter K which is much too large ($K = 545.6 MeV$) for the linear σ-ω model. This deficiency is enhanced by the ETF approximation.

In fig. 1 and 2 we have also included the results of a Hartree calculation for the special case $m_s = 450 MeV$ [30] (this parametrisation is sometimes called PW1). The Hartree results are bracketed by the TF and the ETF data for both t and a_S, being somewhat closer to ETF. Thus ETF overcorrects both a_S and t.

The surface properties of semi-infinite nuclear matter help to understand some of the properties of finite nuclei. For a calculation restricted to symmetric $N = Z = A/2$ nuclei without Coulomb repulsion the energy per particle can be represented by a semi-empirical mass formula

$$\frac{E}{A} - M = a_V + a_S A^{-\frac{1}{3}} + \cdots \tag{43}$$

The large negative volume contribution is compensated in part by the positive surface contribution. The ETF approach with its smaller surface energy a_S leads to nuclei which are more strongly bound than in the Hartree or the TF approximation. As a consequence of this overbinding the nuclei have smaller rms radii in the ETF approximation.

Before turning to the nonlinear case, we want to comment on the fact that we have not been able to solve the ETF equations for semi-infinite nuclear matter using scalar meson masses much larger than $m_s = 500 MeV$. Increasing m_s to 505MeV, we still find a highly accurate solution with, however, a positive slope of the density at the origin. Increasing m_s further leads to unphysical oscillations of the nucleon density which become larger in magnitude with m_s. Finally, for $m_s \approx 510 MeV$ these oscillations do no longer allow an accurate solution of the original variational equations. We believe that this numerical instability has the same origin as the one discussed by Centelles et al. [22], i.e. the $\Delta\phi$-contribution in the source term of the scalar Klein-Gordon equation. The small effective nucleon mass $M_0^*/M = 0.5558$ of the linear σ-ω model confirms the finding of these authors, that the instability occurs whenever M_0^*/M gets smaller than about 0.6. In addition, the problem is also dependent on the value of m_s which enters into the relevant scalar Klein-Gordon equation, i.e. even for $M_0^*/M < 0.6$ one finds values of m_s for which a solution can be obtained. Similar problems occur for some of the nonlinear parametrisations of the effective interaction discussed below. An approach which resolves this problem is presented in Section 4.

3.2 Nonlinear σ-ω model

Extending the original σ-ω model to the nonlinear case offers further degrees of freedom which are commonly used to reproduce additional properties of nuclear matter, e.g. the effective mass at saturation M_0^* and the incompressibility parameter K, which are no longer determined by the saturation properties alone. Since the hypothetical system of infinite nuclear matter is not observable there is some controversy about how to choose M_0^* and K. In the literature M_0^* and K have been considered as parameters which can be used to obtain quantitative agreement with properties of semi-infinite nuclear matter or even with finite nuclei, depending on the physical problem under consideration.

By discussing the semi-infinite system we investigate how surface properties depend on the two additional degrees of freedom. For this analysis we have chosen parametrisations recently proposed by Sharma and Ring [31]. In these parameter sets nuclear matter is required to saturate at $\rho_0 = 0.15 fm^{-3}$ with a binding energy $a_V = -16 MeV$. The incompressibility parameter is significantly reduced to a value $K = 300 MeV$ in accordance with recent experiments on giant monopole resonances [32]. The parametrisations differ in their effective mass which is varied between $M_0^*/M = 0.55$ and $M_0^*/M = 0.80$. Our results for t and a_S as functions of M_0^* are given in fig. 3 and 4 for three different values of m_s. In one case ($m_s = 500 MeV$) a solution of the ETF variational equations has not been possible for $M_0^*/M < 0.6$. t does not only depend on m_s but also on M_0^*. Increasing either of these masses reduces t considerably in both semiclassical approximations. However, the difference $t(TF) - t(ETF)$ shows only little variation with m_s but depends much more strongly on M_0^*. For a small M_0^* this difference is positive in accordance with the result from the linear σ- ω model (compare fig. 1). For a high effective mass the surface thickness is hardly affected by the inclusion of inhomogenity corrections. Even more interesting, fig. 3 indicates that the difference $t(TF) - t(ETF)$ changes its sign for $M_0^*/M \approx 0.79$. The surface energy shows a similar m_s- and M_0^*-dependence as t. The difference $a_s(TF) - a_s(ETF)$ is

positive for a small effective mass in accordance with the linear case but changes sign for large M_0^*. The change of sign can be explained by a comparison of the three second order contributions to the surface energy in the ETF approximation. Whereas the contribution from $(\nabla k_F)^2$ is always positive, the $(\nabla M^*)^2$- and $(\nabla k_F)(\nabla M^*)$-terms give negative contributions. For small effective masses the latter contributions are dominant, thus the net result of the inhomogenity corrections is a lowering of the surface energy. However, as M_0^* is increased, the contributions involving ∇M^* are more and more suppressed leading to the change of sign somewhere around $M_0^*/M = 0.75$. A similar compensation effect has been recognized by Centelles et al.[20] for the case of finite nuclei.

We have not yet discussed the role of the incompressibility parameter K. Decreasing its value softens the nuclear surface. At the same time the surface energy contribution decreases, such that the ratio a_S/t is considerably reduced. The incompressibility is the only parameter which allows to diminish a_S without reducing t. The differences $t(TF)-t(ETF)$ and $a_S(TF)-a_S(ETF)$, however, hardly depend on K as the induced shift in t and a_S is nearly equal for both approximation. This emphasizes once more the great importance of M_0^* for the difference between the TF and the more elaborate ETF approach.

Finally, in table 3 we compare the surface properties of several parametrisations recently used in the literature. The corresponding parameters are listed in table 1 and their nuclear matter properties are given in table 2 in order to allow a direct comparison of their physical implications. Once more we remark that we have not been able to solve the corresponding ETF equations whenever the M_0^* becomes too small. This is the case for the parametrisations NLB2 and SRK3M5 with M_0^*/M around 0.55.

As expected, the differences between the TF and the ETF results are small for parametrisations with large effective masses (SRK3M7). With decreasing M_0^* the semiclassical approximations differ more and more (e.g. for NL2, NLB1 and NL1). It seems that only those parametrisations with rather low M_0^* (of the order $M_0^*/M \simeq 0.6$) are

able to yield acceptable results for both surface properties (NL1, NLB1). Unfortunately it is the range of low effective masses which causes problems in the ETF case.

4 Alternative scheme of solution

In the preceeding section it has been pointed out that it is not possible to solve the ETF variational equations in their original form for some parametrisations. The same difficulties show up in the treatment of finite nuclei (as already recognized by Centelles et al.[22]). As this does not only happen for some extreme parameter sets but rather for some rather successful ones used for the description of nuclear properties (NL1, SRK3M5), it is desirable to find a way to resolve this problem.

Centelles et al.[22] analyzed this problem and found that the coupling of the scalar potential in eq.(17) to a source term which itself contains a $\Delta\phi$ contribution leads to a numerical instability for the critical parametrisations. Since this term originates from the variational derivative of the $(\nabla M^*)^2$ contribution to the effective kinetic energy t_s^*, eq.(14), with respect to ϕ they proposed to attenuate this contribution by introducing a prefactor $\lambda < 1$ into this term which is chosen sufficiently small to allow for a solution of eq.(17). Subsequently the resulting error in the total energy is corrected pertubatively.

Here we suggest a different scheme of solution very much in the spirit of the semiclassical approach. As the semiclassical scalar potential consists of contributions of order $\hbar^0$ as well as $\hbar^2$ it seems reasonable to decompose the scalar potential as

$$\phi = \phi_0 + \phi_2 \tag{44}$$

where the LDA contribution ϕ_0 is defined via

$$(\Delta - m_s^2)\phi_0 = -g_s \sum_{q=p,n} \rho_{sq}^{[0]}[k_q, \phi_0] + b\phi_0^2 + c\phi_0^3. \tag{45}$$

While the exact ϕ_2 consequently satisfies eq.(17) with ϕ replaced via eq.(44) it is now natural to expand the source term of eq.(17) in powers of ϕ_2/ϕ_0 using

$$\rho_{sq}[k_q, \phi] = \rho_{sq}^{[0]}[k_q, \phi_0] + \rho_{sq}^{[2]}[k_q, \phi_0] + \frac{\partial \rho_{sq}^{[0]}[k_q, \phi_0]}{\partial \phi_0}\phi_2 \tag{46}$$

which is very similar to the semiclassical expansion of ρ_{sq} (compare [16]). This expansion leads to a simplified equation for ϕ_2,

$$\begin{aligned}(\Delta - m_s^2)\phi_2 &= -g_s \sum_{q=p,n} \rho_{sq}^{[2]}[k_q, \phi_0] \\ &\quad +(2b\phi_0 + 3c\phi_0^2 - g_s \sum_{q=p,n} \frac{\partial \rho_{sq}^{[0]}[k_q, \phi_0]}{\partial \phi_0})\phi_2. \end{aligned} \tag{47}$$

Since $\rho_{sq}^{[2]}$ only depends on the LDA potential ϕ_0 in this case rather than the complete scalar potential $\phi_0 + \phi_2$ as in the original variational equation (17), this decomposition avoids the coupling of $\Delta\phi$ to itself. For consistency the same expansion is applied to eq.(15) which leads to

$$\frac{k_q}{\epsilon_{q0}} \rho_q^{[2]}[k_q, \phi_0] = \frac{\partial \rho_q}{\partial k_q}(\epsilon_{q0} - \frac{M_0^*}{\epsilon_{q0}}\phi_2 + V_q - \mu_q). \tag{48}$$

The set of eqs.(45,47,48) represents an alternative scheme of solving the ETF variational equations (referred to as ETF2 in the following).

Before we apply this scheme to critical parametrisations we analyze its properties and accuracy using a parameter set for which the original ETF solution is also available. Using the parametrisation NL2 [33], we plot in fig. 5 the contributions to the total scalar potential for the case of ^{208}Pb. Whereas ϕ_2 originates from the solution of eq.(47) the corresponding contribution in the original ϕ is extracted by subtracting the ϕ_0 obtained from the solution of eq.(45). One notes that ϕ_2, which vanishes outside the surface region of the nucleus (as is to be expected), is small compared to the LDA part ϕ_0. The form of the nonlocal contribution ϕ_2 which is determined by the curvature of the nuclear surface is similar to $\phi - \phi_0$ though the extrema are more pronounced in the latter case. In order to demonstrate the accuracy of the ETF2 scheme we compare binding energies and radii of ^{16}O, ^{40}Ca and ^{208}Pb in table 4. One recognizes that the differences between the results of the two schemes are small for all three cases. In particular, the agreement is better in the case of heavy nuclei where the inhomogeneous surface region is small compared to the bulk.

The preceeding discussion has shown that the ETF2 scheme is reliable for parametrisations which are accessible in both schemes. Thus it can be applied with confidence to those parametrisations where the original variational equations can not be solved. As an example we have chosen the parametrisation NL1 [33] which turned out to be very successful for the description of finite nuclei within the relativistic mean field approach (see section 5, table 5).

5 Finite nuclei

In this section we examine to what extent semiclassical methods are able to reproduce Hartree results using identical parametrisations. Once the set of parameters is chosen, all details of the interaction between the nucleons are fixed and our approach here is not to change this interaction, i.e. not to adjust the parameters of QHD to the specific semiclassical approximations used. As gradient expansions can be interpreted as expansions about a homogeneous system using the inhomogenity of the actually system of interest as the "expansion parameter" it has to be examined whether an expansion to second order is sufficient. This is of particular interest, as for the description of nuclei in the nonrelativistic context it turned out to be necessary to include fourth order corrections [7, 8, 9].

From the discussion of semi-infinite nuclear matter in section 3 it is clear that the characteristics of the ETF approximation depend strongly on the parametrisation chosen for the effective interaction. However, parametrisations with very different nuclear matter properties (see table 2) have been suggested (some of them are collected in table 1) using different schemes for their construction. In many cases more or less well established nuclear matter properties are used to fix at least some of the unknown parameters. Sometimes recourse is taken to surface properties of semi-infinite nuclear matter in order to restrict the allowed range of some parameters. Another approach is to optimize the parameters of the effective interaction in order to reproduce properties of finite nuclei such as binding energies, radii and surface thicknesses within the Hartree

approximation [34].

Though we are mainly addressing the question how well Hartree results can be reproduced by semiclassical approximations which, a priori, is independent of the ability of these methods to describe experimental data we base our analysis on parametrisations adjusted to the empirical properties of finite nuclei. In several investigations [34, 15] the parametrisation NL1 turned out to be most successful over the whole range of nuclei in the periodic table for which the variational equations of the ETF approach have to be solved with the ETF2 scheme for this effective interaction due to its small M_0^*. In addition, we have chosen another commonly used parametrisation, NL2 [33], which allows a solution of the original equations. In contrast to NL1 it has the desirable feature of avoiding a negative value of the parameter C in front of the ϕ^4 contribution to the self-coupling of the σ-meson. Consequently, all conceptual problems concerning the asymptotic stability of the scalar Klein-Gordon equation do not apply for this parametrisation [35, 36]. In tables 5 (NL1) and 6 (NL2) we compare the binding energies per particle and the charge radii obtained by the semiclassical approximations for some selected spherical nuclei (distributed over the whole mass range) with the corresponding Hartree results [15]. In case of the energies we have subtracted the center of mass corrections from the Hartree results for comparison. The charge radius r_c is obtained from the proton radius r_p by $r_c = (r_p^2 + 0.64 fm^2)^{1/2}$.

Concerning the ETF approximation one notes a systematic overbinding of all nuclei examined irrespective of the parametrisation chosen. This overbinding is worse in the case of NL2 but even for this parametrisation the deviations from Hartree energies are less than $0.5 MeV/nucleon$ and confirm similar results of nonrelativistic second order calculations using the Skyrme HF approach [6]. In accordance with the overbinding the nuclei are in general too small in the ETF approximation. On the other hand, it is far more difficult to draw definitive conclusions about the behaviour of the TF approximation in this respect: Whereas TF underbinds all nuclei for the parametrisation NL1 it leads to an overbinding in the case of NL2. However, compared to the

ETF approximation the TF binding energies and radii are in closer agreement with the Hartree results for all nuclei in the latter case. At first glance this result seems to question the ETF approach as the inhomogenity corrections worsen the agreement for the special case of NL2. As already discussed by Centelles et al.[22], however, the closer agreement of the TF approximation with Hartree results has to be regarded as fortuitous, caused by the special combination of the interaction parameters. The value $M_0^*/M = 0.67$ in the case of NL2 supports the result of these authors that this close agreement occurs for a value $M_0^*/M \approx 0.68$.

This comparison shows that judging the quality of semiclassical expansions on the basis of just one single effective interaction can be misleading. In the framework of an effective interaction whose parameters can not be definitively determined one has to use quality criteria which are independend of a specific parameter set. One way of doing this is to average over many parametrisations. Rather than collecting parametrisations proposed in the literature we try to take this average in a more systematic way by considering the physically relevant regime in the multidimensional parameter space. However, varying all parameters of the nonlinear σ-ω model seems somewhat excessive and also unnecessary as e.g. the values of the ω and the ρ meson masses are rarely chosen differently from $m_v = 783MeV$ and $m_\rho = 763MeV$. Furthermore, we require the nuclear matter properties already utilized for the discussion of semi-infinite nuclear matter and a typical value of the asymmetry energy parameter a_4,

$$\rho_0 = 0.15fm^{-3} \qquad a_V = -16MeV \qquad a_4 = 35MeV,$$

but allow for free variation of the effective mass, incompressibility and scalar meson mass. These parameters are chosen within the range

$$\begin{aligned} 250MeV &< K < 350MeV \\ 0.6 &< M_0^*/M < 0.75 \\ 450MeV &< m_s < 500MeV \end{aligned}$$

The resulting 8 parametrisations with

$$\begin{aligned}
1 : (K, M_0^*/M, m_s) &= (350, 0.60, 450) \\
2 : (K, M_0^*/M, m_s) &= (350, 0.75, 450) \\
3 : (K, M_0^*/M, m_s) &= (350, 0.75, 500) \\
4 : (K, M_0^*/M, m_s) &= (350, 0.60, 500) \\
5 : (K, M_0^*/M, m_s) &= (250, 0.75, 450) \\
6 : (K, M_0^*/M, m_s) &= (250, 0.60, 450) \\
7 : (K, M_0^*/M, m_s) &= (250, 0.60, 500) \\
8 : (K, M_0^*/M, m_s) &= (250, 0.75, 500)
\end{aligned}$$

may be interpreted as the corner points of a cube which covers the physically interesting range in the K, M_0^*, m_s-space. The corresponding interaction parameters are given in Appendix A. Fig. 6 shows the binding energies per particle obtained from these parametrisations (numbered as above) for the element ^{208}Pb. One would expect that gradient expansion methods which treat the inhomogenity of the actual system as a perturbation to the infinite homogeneous problem are more reliable when applied to a heavy nucleus which consists of an extended bulk region and a comparatively small surface regime where the inhomogenity is concentrated. Since we were not able to solve the original ETF equations for parametrisation 7, this data point has been obtained with the alternative scheme ETF2. The straight lines that connect the data points are only drawn to guide the eye. Concerning the binding energies one recognizes that ETF follows the trend of the Hartree calculations, consistently leading to stronger binding. The TF approximation, on the other hand, depends more sensitively upon changes in the parameters leading to fluctuations around the Hartree result. It underestimates the ETF and even the Hartree binding energies for those parametrisations with low effective masses $M_0^*/M = 0.60$ (1,4,6,7), wheras it overestimates them for high masses $M_0^*/M = 0.75$ (2,3,5,8). Therefore, there are parametrisations for which the difference $E_{TF} - E_H$ vanishes by chance. Looking at the rms charge radii in fig. 7 one recognizes that the

^{208}Pb nucleus is always found to be too large in the TF approximation. The deviation from the Hartree result is largest for those cases with low M_0^* corresponding to an underbound nucleus. The agreement between ETF and Hartree results is much better for all parametrisations under consideration: One observes a modest fluctuation around the Hartree result which, however, can not be attributed to one single parameter.

Repeating this investigation with an enlarged saturation density, $\rho_0 = 0.17 fm^{-3}$, we have verified that these conclusions are independent of the parameter ρ_0.

It must be considered an important advantage of the ETF approach that its quality is much less dependend on the properties of the effective interaction than that of the TF approximation.

6 Conclusions

In this paper we have investigated the properties of relativistic semiclassical approximations for semi-infinite nuclear matter and finite nuclei.

Comparing the TF with the ETF approximation we have studied how second order inhomogenity corrections affect the surface properties of semi-infinite nuclear matter. In the case of the linear σ-ω model we have found a compression of the surface region resulting in a reduced contribution to the surface energy. Turning to the nonlinear σ-ω model with its additional degrees of freedom we have observed a strong dependence of the surface properties on the effective mass M_0^*: Whereas the effect of gradient corrections on surface thickness and energy is similar to the linear situation for small M_0^* a different behaviour is observed for large M_0^* where the inhomogenity corrections tend to soften the surface. This M_0^*-dependence can be explained by a varying degree of cancellation among the different second order contributions. Thus caution is required in drawing definitive conclusions about the effect of second order corrections on the basis of just a single specific effective interaction.

For finite nuclei a comparison is made between the Hartree and the two semiclassical approximations based on the parametrisations NL1 and NL2. Concerning the ETF

approximation we have observed an overbinding of all nuclei which, however, does not exceed 0.5Mev/nucleon. This is in agreement with second order calculations within the Skyrme HF approach [6]. It also has been found that in the case of NL2 the TF energies and radii are closer to Hartree results than the corresponding ETF data. This fortuitous agreement has led us to investigate in more detail the role of the effective interaction for the importance of inhomogeneity corrections by comparing a number of rather different parametrisations probing the physically relevant regime of the complete parameter space. It turned out that on the average ETF results are in better agreement with Hartree results for binding energies and in particular for rms radii. As the TF ground state energies fluctuate around the Hartree energies there exist parametrisations for which they agree by chance. Contrary to the TF approximation the ETF approach leads to a consistent overbinding of ^{208}Pb for *all* parametrisations. Again the situation resembles the nonrelativistic case. This may be a hint that fourth order inhomogenity corrections are necessary to improve the agreement between the semiclassical and the Hartree approach.

Fourth order corrections may be interesting for a further reason: Due to a numerical instability of the scalar Klein-Gordon equation we have not been able to solve the original ETF variational equations for some parametrisations of the effective interaction. Moreover, this problem unfortunately arises for interactions with rather small effective masses M_0^* which seem to provide the most realistic description of finite nuclei. It is worthwhile to study whether this problem is restricted to the second order approximation or persists to fourth order.

In order to overcome this problem which would limit the application of the ETF approach we propose an alternative scheme of solution in section 4. It originates from a semiclassical point of view and thus is consistent with the general concept behind gradient expansions. Its quantitative success for parametrisations for which a comparison with results of the original ETF equations is possible makes it a reliable scheme for extending the range of applicability of the ETF approach. In addition, it allows

for an extension to higher orders in the semiclassical expansion. Due to its internal consistency this scheme could be preferable to the prefactor reduction suggested in Ref.[22].

Finally, it should be emphasized that due to its particle number independence the ETF approach is an ideal tool for the discussion of strongly interacting astrophysical systems (compare [37]).

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Appendix

A Brief review of infinite nuclear matter relations

The effective interaction of the nonlinear σ-ω model is characterized by the dimensionless coupling constants

$$\begin{aligned} C_s^2 &= \frac{g_s^2}{m_s^2}M^2 \qquad C_v^2 = \frac{g_v^2}{m_v^2}M^2 \qquad C_\rho^2 = \frac{g_\rho^2}{m_\rho^2}M^2 \\ B &= \frac{b}{g_s^3}\frac{1}{M} \qquad C = \frac{c}{g_s^4} \end{aligned}$$

and the meson masses m_s, m_v and m_ρ which, however, are irrelevant for infinite nuclear matter. As the five coupling constants determine the saturation density ρ_0, the binding energy a_V, the effective mass $M_0^* \equiv M^*(\rho_0)$ at saturation density, the incompressibility K and the asymmetry energy a_4 of (asymmetric) nuclear matter it is sometimes more appropriate to discuss the effective interaction in terms of these quantities. Therefore, it is necessary to relate the nuclear matter properties to the actual parameters of the interaction. In this appendix we briefly collect the corresponding relations.

The saturation of nuclear matter requires

$$\frac{\epsilon(\rho_0)}{\rho_0} = M + a_V \tag{49}$$

$$P(\rho_0) = 0 \tag{50}$$

where the pressure

$$\begin{aligned} P(\rho) &= \frac{1}{2}\frac{g_v^2}{m_v^2}\rho^2 - \frac{1}{2}\frac{m_s^2}{g_s^2}(M-M^*)^2 - \frac{1}{3}B(M-M^*)^3 - \frac{1}{4}C(M-M^*)^4 \\ &\quad + \frac{4}{3}\int_0^{k_F}\frac{d^3k}{(2\pi)^3}\frac{\underline{k}^2}{(\underline{k}^2 + M^{*2})^{\frac{1}{2}}} \end{aligned} \tag{51}$$

is related to the energy density

$$\begin{aligned} \epsilon(\rho) &= \frac{1}{2}\frac{g_v^2}{m_v^2}\rho^2 + \frac{1}{2}\frac{m_s^2}{g_s^2}(M-M^*)^2 + \frac{1}{3}B(M-M^*)^3 + \frac{1}{4}C(M-M^*)^4 \\ &\quad + 4\int_0^{k_F}\frac{d^3k}{(2\pi)^3}(\underline{k}^2 + M^{*2})^{\frac{1}{2}} \end{aligned} \tag{52}$$

by

$$P = \rho^2 \frac{\partial}{\partial \rho}\left(\frac{\epsilon}{\rho}\right) = \rho \frac{\partial \epsilon}{\partial \rho} - \epsilon. \tag{53}$$

Since the pressure vanishes at the saturation density ρ_0 one has

$$\frac{\epsilon(\rho_0)}{\rho_0} = \frac{\partial \epsilon}{\partial \rho}(\rho_0) = \frac{g_v^2}{m_v^2}\rho_0 + \sqrt{k_{F0}^2 + M_0^{*2}} \tag{54}$$

and eq.(49) can be replaced by

$$\frac{g_v^2}{m_v^2}\rho_0 + \sqrt{k_{F0}^2 + M_0^{*2}} = M + a_V. \tag{55}$$

The effective mass as a function of the density is defined by the implicit equation

$$M^* = M - \frac{g_s^2}{m_s^2}\left(\rho_s(k_F, M^*) - B(M - M^*)^2 - C(M - M^*)^3\right) \tag{56}$$

where the scalar density is given by

$$\rho_s(k_F, M^*) = \frac{M^*}{2\pi^2}(k_F \epsilon_F - M^{*2} \ln \frac{k_F + \epsilon_F}{M^*}).$$

The incompressibility of nuclear matter is

$$\begin{aligned} \frac{1}{9}K &= \rho^2 \frac{\partial^2}{\partial \rho^2}\left(\frac{\epsilon}{\rho}\right)\Bigg|_{\rho=\rho_0} \\ &= \frac{g_v^2}{m_v^2}\rho + \frac{1}{3}\frac{k_F^2}{\epsilon_F} + \frac{M^{*2}}{\epsilon_F^2}\rho \\ &\quad \cdot \left[3(\frac{\rho}{\epsilon_F} - \frac{\rho_s}{M^*}) - \frac{m_s^2}{g_s^2} - 2B(M - M^*) - 3C(M - M^*)^2\right]^{-1}\Bigg|_{\rho=\rho_0} \end{aligned} \tag{57}$$

and the coefficient of the asymmetry energy is usually defined as

$$a_4 = \frac{1}{6}\frac{k_F^2}{\epsilon_F} + \frac{1}{8}\frac{g_\rho^2}{m_\rho^2}\rho\Big|_{\rho=\rho_0}. \tag{58}$$

The system of equations (50,55-58) can be solved analytically. Denoting the integral contribution to the pressure, eq.(51), by p the dimensionless coupling constants can be written as

$$\frac{C_v^2}{M^2} = \frac{1}{\rho}(M + a_V - \epsilon_F)\Big|_{\rho=\rho_0, M^*=M_0^*} \tag{59}$$

$$\frac{M^2}{C_s^2}(M-M^*)^2 = 6\frac{C_v^2}{M^2}\rho^2 + 3(\frac{\rho}{\epsilon_F} - \frac{\rho_s}{M^*})(M-M^*)^2 - 6\rho_s(M-M^*) \quad (60)$$
$$+12p + \frac{M^{*2}}{\epsilon_F^2}\rho(M-M^*)^2\left[\frac{C_v^2}{M^2}\rho + \frac{1}{3}\frac{k_F^2}{\epsilon_F} - \frac{1}{9}K\right]^{-1}\Bigg|_{\rho=\rho_0, M^*=M_0^*}$$

$$\frac{1}{3}B(M-M^*)^3 = -4\frac{C_v^2}{M^2}\rho^2 - 3(\frac{\rho}{\epsilon_F} - \frac{\rho_s}{M^*})(M-M^*)^2 + 5\rho_s(M-M^*) \quad (61)$$
$$-8p - \frac{M^{*2}}{\epsilon_F^2}\rho(M-M^*)^2\left[\frac{C_v^2}{M^2}\rho + \frac{1}{3}\frac{k_F^2}{\epsilon_F} - \frac{1}{9}K\right]^{-1}\Bigg|_{\rho=\rho_0, M^*=M_0^*}$$

$$\frac{1}{4}C(M-M^*)^4 = \frac{3}{2}\frac{C_v^2}{M^2}\rho^2 + \frac{3}{2}(\frac{\rho}{\epsilon_F} - \frac{\rho_s}{M^*})(M-M^*)^2 - 2\rho_s(M-M^*) \quad (62)$$
$$+3p + \frac{1}{2}\frac{M^{*2}}{\epsilon_F^2}\rho(M-M^*)^2\left[\frac{C_v^2}{M^2}\rho + \frac{1}{3}\frac{k_F^2}{\epsilon_F} - \frac{1}{9}K\right]^{-1}\Bigg|_{\rho=\rho_0, M^*=M_0^*}$$

$$\frac{C_\rho^2}{M^2} = \frac{8}{\rho}(a_4 - \frac{1}{6}\frac{k_F^2}{\epsilon_F})\Bigg|_{\rho=\rho_0, M^*=M_0^*}. \quad (63)$$

In table 7 we list the dimensionless coupling constants of the eight parametrisations introduced in Section 5.

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Table captions

Table 1 Parameters of some effective forces discussed in the literature. See Appendix A for the definition of the parameters. All masses are in MeV.

Table 2 Nuclear matter properties ρ_0, a_V, M^*/M and K of the effective forces considered in table 1.

Table 3 Surface properties a_S and t of semi-infinite nuclear matter in TF and ETF approximation for the effective forces of table 1.

Table 4 Comparison of binding energies E and r.m.s. charge radii r_c of some nuclei in ETF and ETF2 approximation for the parametrisation NL2.

Table 5 Binding energies E/A and r.m.s. charge radii r_c of some selected nuclei in TF, ETF and Hartree approximation with the force NL1. The Hartree results are taken from [15]. The center of mass correction $E_{cm} = -30.75A^{-1/3}MeV$ has been subtracted from the Hartree energies.

Table 6 Same as table 5 with the force NL2.

Table 7 Dimensionless coupling constants of the 8 parametrisations discussed in Section 5.

Table 1

	C_s^2	C_v^2	C_ρ^2	$B \cdot 10^3$	$C \cdot 10^3$	m_s	m_v	m_ρ	M
PW1 [30]	267.568	196.300				450	783		939
NLB1 [38]	305.53	200.00	83.56	2.353	-2.857	470	783	770	938.9
NLB2 [38]	353.89	238.58	73.94	2.165	-3.142	485	783	770	938.9
NL1 [34]	373.176	245.458	149.67	2.4578	-3.4334	492.25	795.36	763	938
NL2 [33]	286.513	191.022	183.336	0.6409	2.0002	504.89	780	763	938
SRK3M5 [31]	380.792	264.687	26.645	1.618	-2.297	500	783	763	939
SRK3M5 [31]	233.239	132.497	85.645	3.292	3.978	500	783	763	939

Table 2

	ρ_0 $[fm^{-3}]$	a_V $[MeV]$	M^*/M	K $[MeV]$
PW1	0.1934	-15.75	0.556	546
NLB1	0.1625	-15.75	0.621	280
NLB2	0.1625	-15.75	0.556	245
NL1	0.1518	-16.42	0.573	211
NL2	0.1456	-17.02	0.670	399
SRK3M5	0.15	-16	0.55	300
SRK3M7	0.15	-16	0.75	300

Table 3

	a_S $[MeV]$		t $[fm]$	
	TF	ETF	TF	ETF
PW1	35.90	32.85	2.71	2.27
NLB1	22.32	20.77	2.66	2.19
NLB2	21.78		3.00	
NL1	19.79	17.39	2.90	2.05
NL2	20.84	19.65	1.81	1.51
SRK3M5	21.61		2.70	
SRK3M7	16.03	16.24	1.59	1.47

Table 4

	E $[MeV]$		r_c $[fm]$	
	ETF	ETF2	ETF	ETF2
^{16}O	-118.62	-118.60	2.578	2.584
^{40}Ca	-349.57	-349.54	3.336	3.340
^{208}Pb	-1680.81	-1680.77	5.507	5.508

Table 5

	E/A $[MeV]$			r_c $[fm]$		
	H	ETF2	TF	H	ETF2	TF
^{16}O	-7.18	-7.49	-7.03	2.77	2.64	2.80
^{56}Ni	-8.47	-8.75	-8.31	3.70	3.73	3.89
^{90}Zr	-8.68	-8.92	-8.56	4.28	4.23	4.39
^{118}Sn	-8.57	-8.74	-8.42	4.64	4.60	4.75
^{136}Xe	-8.36	-8.51	-8.24	4.82	4.80	5.14
^{140}Ce	-8.39	-8.56	-8.27	4.90	4.86	5.00
^{208}Pb	-7.83	-7.97	-7.74	5.53	5.56	5.68

Table 6

	$E/A\,[MeV]$			$r_c\,[fm]$		
	H	ETF	TF	H	ETF	TF
^{16}O	-7.14	-7.41	-7.21	2.70	2.58	2.63
^{56}Ni	-8.36	-8.89	-8.68	3.76	3.69	3.74
^{90}Zr	-8.70	-9.08	-8.90	4.27	4.21	4.26
^{118}Sn	-8.61	-8.89	-8.72	4.62	4.58	4.62
^{136}Xe	-8.36	-8.60	-8.45	4.80	4.77	4.81
^{140}Ce	-8.46	-8.70	-8.55	4.88	4.84	4.87
^{208}Pb	-7.83	-8.08	-7.95	5.50	5.51	5.54

Table 7

n	C_s^2	C_v^2	C_ρ^2	$B \cdot 10^3$	$C \cdot 10^3$
1	335.5838	232.2332	105.0646	1.319	-1.398
2	221.2864	132.4772	124.0442	0.518	11.382
3	221.2864	132.4772	124.0442	0.518	11.382
4	335.5838	232.2332	105.0646	1.319	-1.398
5	245.6065	132.4772	124.0442	5.887	-2.937
6	350.7005	232.2332	105.0646	2.282	-3.004
7	350.7005	232.2332	105.0646	2.282	-3.004
8	245.6065	132.4772	124.0442	5.887	-2.937

Figure captions

Fig. 1 Surface thickness t as a function of the scalar meson mass m_s in the TF (dashed) and the ETF (solid line) approximation. For comparison the result of one Hartree calculation (+) is included [30].

Fig. 2 Same as fig. 1 for the surface energy a_s.

Fig. 3 Surface thickness t as a function of M^*/M in the TF (dashed) and the ETF (solid line) approximation for three different scalar meson masses.

Fig. 4 Same as fig. 3 for the surface energy a_s.

Fig. 5 Different contributions to the scalar potential of ^{208}Pb with the force NL2: $0.05 * \phi_0$ (solid line), ϕ_2 (dashed-dotted line) and $\phi - \phi_0$ (dashed line).

Fig. 6 Binding energies per nucleon E/A of ^{208}Pb for 8 selected parametrisations ($n = 1, \ldots, 8$; see table 7) in the TF (open circles), ETF (crossed circles) and Hartree (full circles) approximations.

Fig. 7 Same as fig. 6 for the r.m.s. charge radius r_c of ^{208}Pb.