Theorieteil 1

1.1. a:

$$\vec{p} = \int d^3r \, \rho(\vec{r}) \, \vec{r}$$

1.1. b)

$$\vec{p} = \int d^3r \left[q_1 \delta(\vec{r} - \vec{r}_1) \vec{r}_1 + q_2 \delta(\vec{r} - \vec{r}_2) \vec{r}_2 \right]$$

$$= q_1 \vec{r}_1 + q_2 \vec{r}_2 = q(2,2,2) - q(1,1,1)$$

$$= q(1,1,1) = \left(\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}\right) C\,l.$$

1.1. c) S. 79 Skript

$$E = K \frac{\frac{1}{2} \cdot \left(-\frac{1}{2}\right)}{d} \qquad \left(\text{allgemein: } K \frac{q_1 q_2}{d}\right)$$

$$= -\frac{K}{4} \cdot \frac{1}{d} \, ; \qquad d = \sqrt{(2-1)^2 + (2-1)^2 + (2-1)^2} = \sqrt{3}$$

Ergo:

$$\boxed{E = -\frac{K}{4\sqrt{3}}}$$

1.1. d) Es ist notwendig zu renormieren, weil die Elektrostatische Energie eines Punktteilchens unendlich ist.

Weg 1:

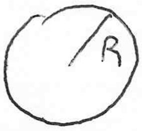
$$E_{feld} = K \int d^3r\, d^3r' \frac{\rho(\vec{r})\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}$$

Für $\rho(\vec{r}) = q\,\delta(\vec{r}-\vec{r}_0)$ hat man

$$E_{fel} = K \int d^3r\, d^3r' \frac{\delta(\vec{r}-\vec{r}_0)\,\delta(\vec{r}'-\vec{r}_0)}{|\vec{r}-\vec{r}'|} =$$

$$= K \int d^3r \frac{\delta(\vec{r}-\vec{r}_0)}{|\vec{r}-\vec{r}_0|} = \frac{K}{|\vec{r}_0-\vec{r}_0|} = +\infty.$$

Weg 2: Kugel mit Radius R



$$\vec{E} = \frac{KQ}{r^2}\frac{\vec{r}}{r} \quad r \geq R, \ (0 \text{ für } r < R)$$

$$w = \frac{K^2 Q^2}{r^4}$$

$$E_{fel} = \frac{\varepsilon_0}{2}\int_R^\infty \frac{K^2 Q^2}{r^4}\,dr \cdot 4\pi r^2 = \cancel{\frac{\varepsilon_0 K^2 Q^2}{2}\left(\frac{r^{-3}}{-3}\right)\Big|_R^\infty = \dots}$$

$$\cancel{= \frac{\varepsilon_0 K^2 Q^2}{6} R^{-3} = +\infty \text{ für } R \mapsto 0}.$$

$$= \frac{\varepsilon_0}{2} K^2 Q^2 4\pi \int_R^\infty \frac{1}{r^2}\,dr =$$

$$= \frac{\varepsilon_0 K^2 Q^2}{2} 4\pi \left(\frac{r^{-1}}{-1}\right)\Big|_R^\infty = \frac{\varepsilon_0}{2} K^2 Q^2 4\pi \frac{1}{R} \longmapsto +\infty \text{ für } R \mapsto 0$$

a)

$\rho(t,\vec{r}) = q\,\delta(z-vt)\,\delta(x)\,\delta(y)$ 2P. $q\checkmark$ δ-Fkt.

b) $\vec{J} = (0, 0, q\cdot v\cdot\delta(z-vt)\,\delta(x)\,\delta(y))$ 2P. $\vec{v}\checkmark \cdot \rho$

c) $\vec{m} = \frac{1}{2}\int d^3r\, \vec{r}\times\vec{J}(\vec{r})$ 1P.

d) 4P.

$$\vec{r}\times\vec{J} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ 0 & 0 & J_z \end{vmatrix} =$$

$= \vec{i}\,(y J_z) - \vec{j}\,(x J_z)$. 1P.

Ergo:

$$m_x = \frac{1}{2}\int_{-\infty}^{\infty} dx\,dy\,dz\; y\,q\,v\,\delta(z-vt)\,\delta(x)\,\delta(y)$$

$$= \frac{qv}{2}\underbrace{\int_{-\infty}^{\infty} dx\,\delta(x)}_{1}\;\underbrace{\int_{-\infty}^{\infty} y\,\delta(y)\,dy}_{=0}\;\underbrace{\int_{-\infty}^{\infty} dz\,\delta(z-vt)}_{=1} = 0$$

je 1P. m_x, m_y, m_z

Nämlich: $\int_{-\infty}^{\infty} y\,\delta(y)\,dy = 0$ $\left(\int f(y)\,\delta(y)\,dy = f(0) = 0\right.$ in diesem Fall, da $f(y) = y$

Genauso:

$$m_y = -\frac{qv}{2} \int_{-\infty}^{\infty} dx\,dy\,dz \; x\, \delta(z-vt)\,\delta(x)\,\delta(y) = 0,$$

weil

$$\int_{-\infty}^{\infty} x\,\delta(x)\,dx = 0 .$$

Insgesamt:

$$\vec{m} = (0, 0, 0) .$$