NON EXPONENTIAL DECAY and ZENO EFFECT

Consider the unstable state $|S\rangle$ at time $t=0$.

The amplitude that $|S\rangle$ did not decay at the instant t is given by

$$a(t) = \langle S| e^{-iHt} |S\rangle$$

$$\left(|\text{state}(t)\rangle = \underbrace{e^{-iHt}}_{\text{time-evolution-operator}} |S\rangle \right)$$

$$p(t) = |a(t)|^2$$ is the survival probability, i.e. the probability that the state did not decay at the instant t.

$p(0) = 1$ by construction.

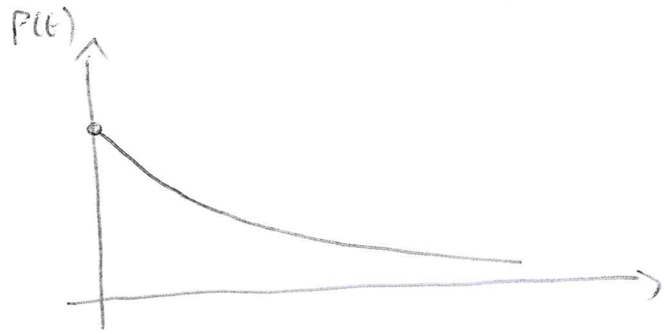
Usually one writes

$$p(t) = e^{-\Gamma t}$$

Γ = decay width ; $\Gamma = \frac{1}{\tau}$ → τ = life-time

However, this is only an approximation!

$P(t) = e^{-\Gamma t}$, $P'(t) = -\Gamma e^{-\Gamma t} \longrightarrow P'(0) = -\Gamma$



Let us show that this is <u>not</u> possible.

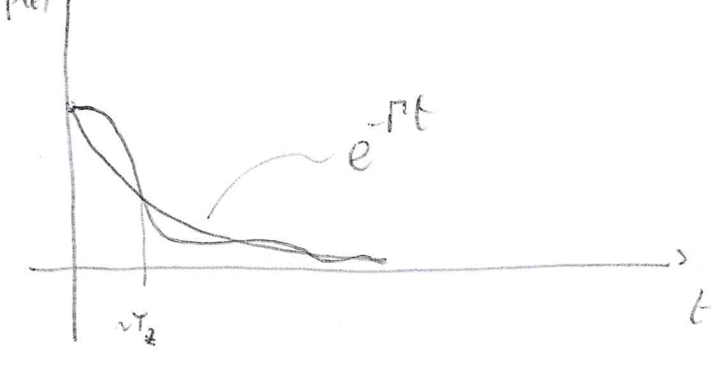
$$a(t) = \langle S | e^{-iHt} | S \rangle = \langle S | 1 - iHt - \frac{H^2}{2} t^2 + \dots | S \rangle =$$

$$= 1 - it \langle S | H | S \rangle - \frac{t^2}{2} \langle S | H^2 | S \rangle + \dots$$

$$a^*(t) = 1 + it \langle S | H | S \rangle - \frac{t^2}{2} \langle S | H^2 | S \rangle + \dots$$

$$P(t) = a^*(t) a(t) = |a(t)|^2 = 1 - \cancel{it \langle S | H | S \rangle} + \cancel{it \langle S | H | S \rangle}$$
$$- t^2 \left(\langle S | H^2 | S \rangle - \langle S | H | S \rangle^2 \right)$$

$$\rightsquigarrow \quad P(t) = 1 - \frac{t^2}{\tau_z^2} + \dots \longrightarrow P'(t) = -\frac{2t}{\tau_z^2} + \dots \longrightarrow P'(0) = 0 \,!$$



The exponential function is only a 'limit'.

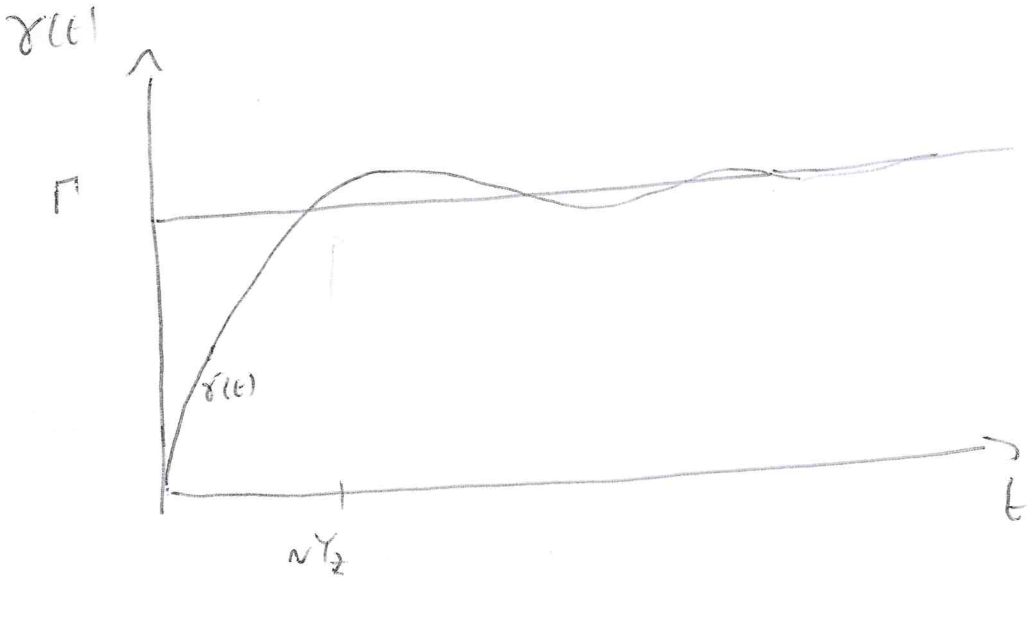
(Indeed, for very large t there are also deviations from the exp-law ... so, for both short and large t the exp-law does not work).

The non-exponential behaviour has been confirmed by recent experiment (1990, 1996, ...)

In general, we can write p(t) as follows

$$p(t) = e^{-\gamma(t)t}$$

$$\begin{cases} p'(t) = -\left(\gamma'(t)t + \gamma(t)\right) e^{-\gamma(t)t} \\ p'(0) = -\gamma(0) e^{-\gamma(0)t} = 0 \rightarrow \gamma(0) = 0 \end{cases}$$



Now, let us come to the zero effect.

Suppose we make one measurement at the instant $T >> \tau_Z$.

$\gamma(T) \simeq \Gamma$.

$P(T) = e^{-\gamma(T)T} \doteq e^{-\Gamma T}$ is the prob. that $|S\rangle$ did not decay.

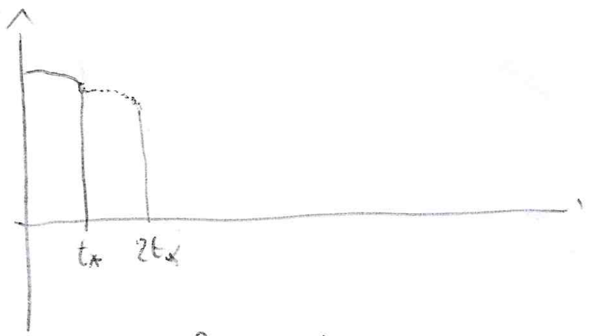
Suppose now that, instead, we perform N measurements at time t_* such that

$$T = N t_*$$

Which is the probability that the state $|S\rangle$ still did not decay after N steps?

After the first step, $t = t_*$

$p(t_*) = e^{-\gamma(t_*)t_*}$ is the prob. that it did not decay. But then, the state collapses back to $|S\rangle$ (collapse of the wave function!).



Then, everything starts from the very beginning. The prob. that after 2 steps $|S\rangle$ did not decay reads

$$P(t_*) \cdot P(t_*) = P(t_*)^2.$$

Similarly, after N steps

$$P(t_*)^N = \left(e^{-\gamma(t_*)t_*}\right)^N = e^{-\gamma(t_*)\overbrace{Nt_*}^{T}} = e^{-\gamma(t_*)T}$$

(to be compared with $P(T) = e^{-\Gamma T}$)

Now, if $\gamma(t) = \Gamma\ \forall t$, we would find again that the prob. is $e^{-\Gamma T}$.

So, in the purely exp case nothing changes.

How, the exp result is wrong and $\gamma(t)$ is not a constant.

Indeed, $\gamma(t_*)$ can be chosen as small as desired!

(it is enough to decrease t_*, i.e. to increase N).

Then:

$$P(t_*)^N = e^{-\gamma(t_*)T} \simeq 1 \quad \text{for } t_* \ll \tau_Z\,!$$

Ergo, the particle does <u>not</u> decay: QUANTUM-ZENO-EFFEKT.

Verified experimentally!

"Watched pot does not boil"

Why the name Zero?

Zero of Elea denied the concept of movement.

Heraclitus / Parmenides

Zero

Turtle

arrow