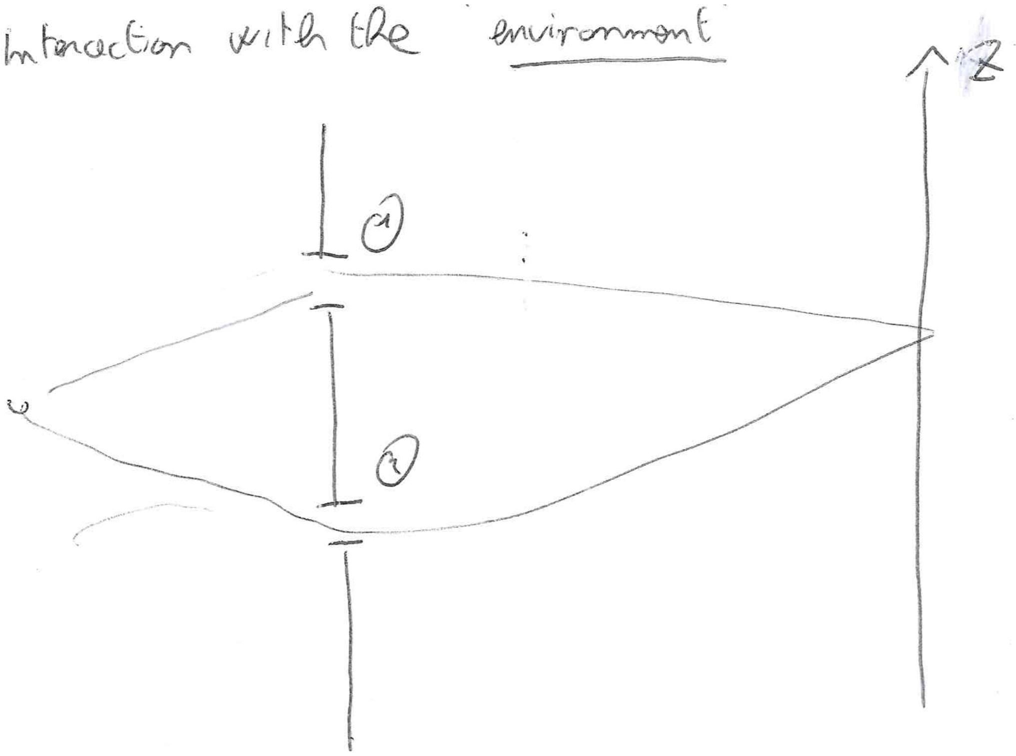
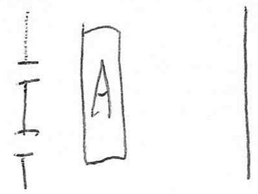
Interaction with the <u>environment</u>



$$\psi = \frac{1}{\sqrt{2}}\left(\psi_1(z) + \psi_2(z)\right)$$

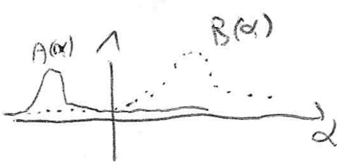
$$\rho(z) = \psi^*(z)\psi(z) = \frac{1}{2}|\psi_1(z)|^2 + \frac{1}{2}|\psi_2(z)|^2 + \underbrace{\frac{1}{2}\psi_1^*(z)\psi_2(z) + \frac{1}{2}\psi_1^*(z)\psi_2(\ldots}_{\text{This term is crucial for the interference...}}$$

Let us now put an apparatus that measure if the particle went through 1 or 2...



$$\psi_{tot}(z, \vec{\alpha}) = \frac{1}{\sqrt{2}}\left(\psi_1(z)A(\vec{\alpha}) + \psi_2(z)B(\vec{\alpha})\right)$$

Now, $A(\vec{\alpha})\cdot B(\vec{\alpha}) \simeq 0$



It follows that

$$|\psi_{tot}(z,\vec{\alpha})|^2 = \frac{1}{2}|\psi_1(z)|^2|A(\vec{\alpha})|^2 + \frac{1}{2}|\psi_2(z)|^2|B(\vec{\alpha})|^2$$

$$+ \underbrace{\frac{1}{2}\psi_1^*\psi_2 A^*(\vec{\alpha})B(\vec{\alpha}) + \frac{1}{2}\psi_1\psi_2^* A(\vec{\alpha})B^*(\vec{\alpha})}_{\simeq 0}$$

$$= \frac{1}{2}|\psi_1(z)|^2|A(\vec{\alpha})|^2 + \frac{1}{2}|\psi_2(z)|^2|B(\vec{\alpha})|^2$$

Integrating over $\vec{\alpha}$:

$$\int d\vec{\alpha}\, |\psi_{tot}(z,\vec{\alpha})|^2 = \frac{1}{2}|\psi_1(z)|^2 + \frac{1}{2}|\psi_2(z)|^2$$

$\longrightarrow$ NO INTERFERENCE IS PRESENT!

In general, decoherence generates a dephasing

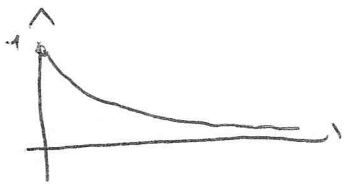
$$|CAT\rangle = \sqrt{\frac{1}{2}}\left(|L\rangle + |D\rangle\right)$$

$$|CAT\rangle = \sqrt{\frac{1}{2}}\left(|L\rangle + e^{i\varphi(t)}|D\rangle\right)$$

$$\rho = \begin{pmatrix} \frac{1}{2} & \frac{1}{2}e^{i\varphi(t)} \\ \frac{1}{2}e^{-i\varphi(t)} & \frac{1}{2} \end{pmatrix}$$

$\varphi(t)$ "random phase"

$$\langle e^{i\varphi(t)} \rangle = e^{-\lambda t}$$



$$\lambda = \frac{1}{2}\langle \varphi^2(t) \rangle .$$

The average of the phase contribution goes to zero.

Later we will see it on a "simple model" ....

Simple Model for Decoherence

Let us consider a single atom with spin $1/2$, which is described by the state

$$\boxed{|S\rangle = a|+\rangle + b|-\rangle .} \qquad |a|^2+|b|^2=1$$

This is obviously a superposition.

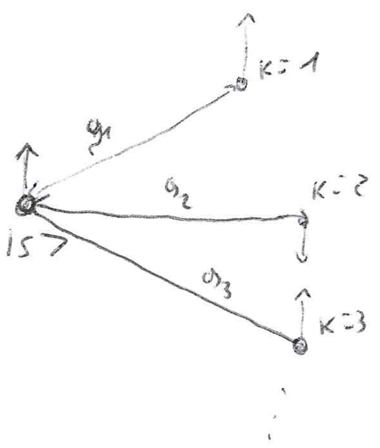
Let us now consider N atoms, which interact with our system. The interaction hamiltonian is supposed to be

acts on the 1st neighbouring atom

acts on the 2nd atom...

$$H = \hbar g_1 \sigma_z \sigma_z^{(1)} + \hbar g_2 \sigma_z \sigma_z^{(2)} + \dots$$

$$= \hbar \sum_K g_K \sigma_z \sigma_z^{(K)}$$



N interactions.

Recall :

The statistical operator of the system S is given by

$$\varrho_S = |S\rangle\langle S| = (a|+\rangle + b|-\rangle)(a^*\langle +| + b^*\langle -|)$$

$$= |+\rangle\langle +|\,|a|^2 + |-\rangle\langle -|\,|b|^2$$

$$+ ab^*|-\rangle\langle +| + a^*b|+\rangle\langle -|$$

$$= (|+\rangle, |-\rangle) \underbrace{\begin{pmatrix} |a|^2 & a^*b \\ ab^* & |b|^2 \end{pmatrix}}_{\varrho_{ij}} \begin{pmatrix} \langle +| \\ \langle -| \end{pmatrix}$$

Nb: $\mathrm{Tr}\,\varrho = |a|^2 + |b|^2 = 1$

However, $\varrho^2 = \begin{pmatrix} |a|^2 & a^*b \\ ab^* & |b|^2 \end{pmatrix}\begin{pmatrix} |a|^2 & a^*b \\ ab^* & |b|^2 \end{pmatrix}$

$\mathrm{Tr}\,\varrho^2 \leq 1$ -> we do not have a pure state.

system

$$|S\rangle_{t=0} = a|+\rangle + b|-\rangle \qquad \text{at } t=0$$

$$|E\rangle_{t=0} = \prod_{K=1}^{N}\left(\alpha_K|+\rangle_K + \beta_K|-\rangle_K\right) \qquad \text{at } t=0$$

Environment

$$|\Psi\rangle_{t=0} = |S\rangle_{t=0}|E\rangle_{t=0}$$

Let us now calculate the evolution in time.

$$U(t,0) = e^{-iHt}$$

$$\begin{cases} \sigma_z|+\rangle = |+\rangle, \quad \sigma_z|-\rangle = |-\rangle \\ \sigma_z^{(K)}|+\rangle_K = |+\rangle_K, \quad \sigma_z|-\rangle_K = -|-\rangle_K. \end{cases}$$

$$|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle \qquad e^{-iHt} = e^{-iH_1t}\cdot e^{-iH_2t}\cdots e^{-iH_Nt}$$

$$H_K = g\sigma_z\sigma_z^{(K)}$$

$$\boxed{\begin{aligned}|\Psi(t)\rangle = {} & a|+\rangle\prod_{K=1}^{N}\left(\alpha_K e^{-ig_Kt}|+\rangle_K + \beta_K e^{ig_Kt}|-\rangle_K\right) \\ & + b|-\rangle\prod_{K=1}^{N}\left(\alpha_K e^{ig_Kt}|+\rangle_K + \beta_K e^{-ig_Kt}|-\rangle_K\right)\end{aligned}}$$

$|\psi(t)\rangle\langle\psi(t)|$ is the <u>full</u> density operator.

It is a <u>complicated</u> object.

$\rho = |\psi(t)\rangle\langle\psi(t)|$ is actually a <u>$2^{N+1}\cdot 2^{N+1}$ matrix!</u>

Let us evaluate it in the case $N=1$:

$$|\psi(t)\rangle = \underline{a|+\rangle}\left(\alpha_1 e^{-ig_1 t}|+\rangle_1 + \beta_1 e^{ig_1 t}|-\rangle_1\right)$$
$$+\, \underline{b|-\rangle}\left(\alpha_1 e^{ig_1 t}|+\rangle_1 + \beta_1 e^{-ig_1 t}|-\rangle_1\right)$$

$$\langle\psi(t)| = \underline{a^*\langle+|}\left(\alpha_1^* e^{+ig_1 t}\langle+|_1 + \beta_1^*\langle-|_1 e^{-ig_1 t}\right)$$
$$+\, \underline{b^*\langle-|}\left(\alpha_1^* e^{-ig_1 t}\langle+|_1 + \beta_1^*\langle-|_1 e^{ig_1 t}\right);$$

$$|\psi(t)\rangle\langle\psi(t)| = |\alpha_1|^2|a|^2\, |+\rangle|+\rangle_1\langle+|\langle+|_1 + |a|^2 e^{-2ig_1 t}|+\rangle|+\rangle_1\langle+|\langle-|_1\, \alpha_1\beta_1^*$$
$$+\, |a|^2\, |+\rangle|-\rangle_1\langle+|\langle+|_1 e^{2ig_1 t} + \ldots$$

$$= \left(|+\rangle|+\rangle_1,\ |+\rangle|-\rangle_1,\ |-\rangle|+\rangle_1,\ |-\rangle|-\rangle_1\right)\begin{pmatrix} \underline{\underline{|a|^2|\alpha_1|^2}} & |a|^2 e^{-2ig_1 t} & \cdots & \\ \cdots & \underline{\underline{|a|^2|\beta_1|^2}} & \cdots & \\ \cdots & \cdots & \underline{\underline{|b|^2|\alpha_1|^2}} & \cdots \\ \cdots & \cdots & \cdots & \underline{\underline{|b|^2|\beta_1|^2}} \end{pmatrix}\begin{pmatrix} \langle+|\langle+|_1 \\ \langle+|\langle-|_1 \\ \langle-|\langle+|_1 \\ \langle-|\langle-|_1 \end{pmatrix}$$

<u>4×4 Matrix</u>

We now introduce the reduced density operator

$$\rho_{red} = \underset{\text{over } N \text{ atoms}}{\mathrm{Tr}} [\rho] =$$

$$= \sum_{i_1 \dots i_N = \pm 1} \langle s_1 \dots s_N | \rho | s_1 \dots s_N \rangle ;$$

In our example ($N=1$):

$$\rho_{red} = {}_1\langle + | \rho | + \rangle_1 + {}_1\langle - | \rho | - \rangle_1 =$$

$$= |\alpha_1|^2 |a|^2 |+\rangle\langle+| + |b|^2 |\alpha_1|^2 |-\rangle\langle-| \dots =$$

$$= |a|^2 |+\rangle\langle+| + |b|^2 |-\rangle\langle-| + z(t)\, a b^* |+\rangle\langle-| + z^*(t)\, a^* b |-\rangle\langle+|$$

whereas:

$$\boxed{z(t) = \cos(2g_1 t) + i\left(|\alpha_1|^2 - |\beta_1|^2\right)\sin(2g_1 t);}$$

Now, when going to the N-atom case we get the same with

$$z(t) = \prod_{k=1}^{N} \left[\cos(2g_k t) + i\left(|\alpha_k|^2 - |\beta_k|^2\right) \sin(2g_k t) \right]$$

$z(t)$ depends on $|\alpha_k|^2$ and $|\beta_k|^2$

$t = 0$

$z(0) = 1$

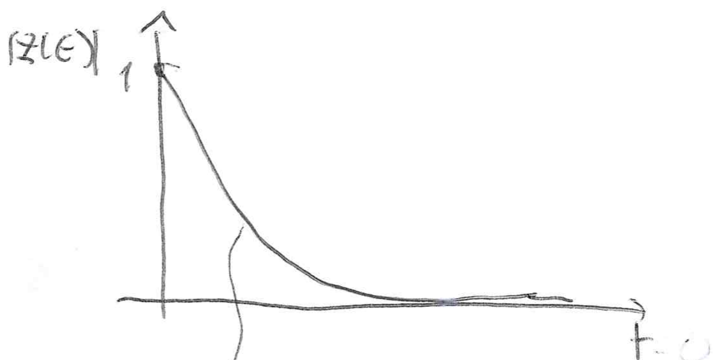
$$|z_k| = \sqrt{\cos^2(2g_k t) + \left(|\alpha_k|^2 - |\beta_k|^2\right)^2 \sin^2(2g_k t)}$$

Now, $\left(|\alpha_k|^2 - |\beta_k|^2\right)^2 \leq 1$ and generally < 1

$$|z| = \prod_k |z_k| \longmapsto 0$$ for $t > 0$ very fast...

We then see decoherence...

$$z(t) = |z(t)|\, e^{i \varphi_z(t)}$$ with



faster and faster for increasing N.

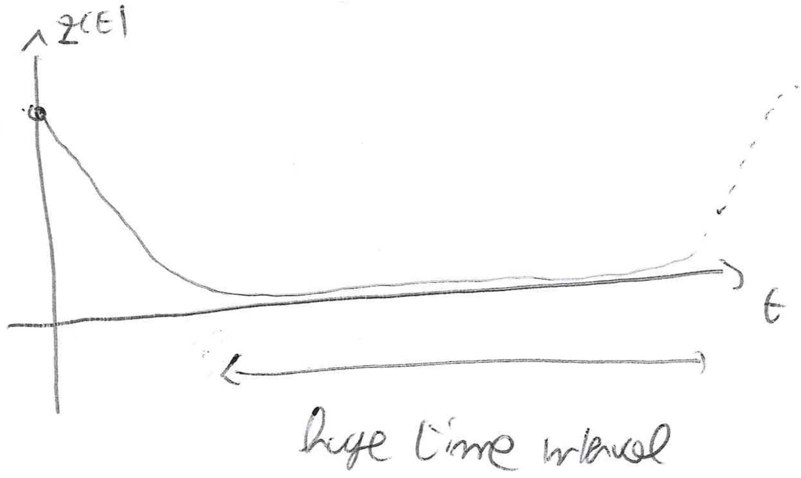
For the reduced operator we then get

$$\text{Matrixform}(\rho_{red}) = \begin{pmatrix} |a|^2 & \overset{\simeq 0}{z(t)ab^*} \\ z^*(t)a^*b & |b|^2 \end{pmatrix}$$

This fact is general to every decoherence model.

How,

$z(t)$ will come back (Poincaré time)



(This is anyhow a mathematical theorem!)

The solution of decoherence is a FAPP solution!

How, the superposition is still there!!!