Computational Methods for Kinetic Processes in Plasma Physics

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* Plasma physics on computer (General description)
* Kinetic Plasma Simulations for high energy particles
* How PIC works (cold plasma dispersion, plasma dispersion function)
* Electrostatic codes (grid quantities, beat heating)
* Finite-difference Time-Domain Maxwell solver on Yee grid
* Particle movers: Boris's algorism
* Conservative charge deposition method
* Boundary conditions (particles and fields)
* Simulations for astrophysical plasmas
* Recent work 1: Weibel instability in relativistic jets (radiation, weighted beam)
* Recent work 2: Kinetic Kelvin-Helmholtz instability (particle acceleration)
* Run MPI-Tristan code for global jet
* Analyze the simulation result using NCARGraphic, VisIt and other tools

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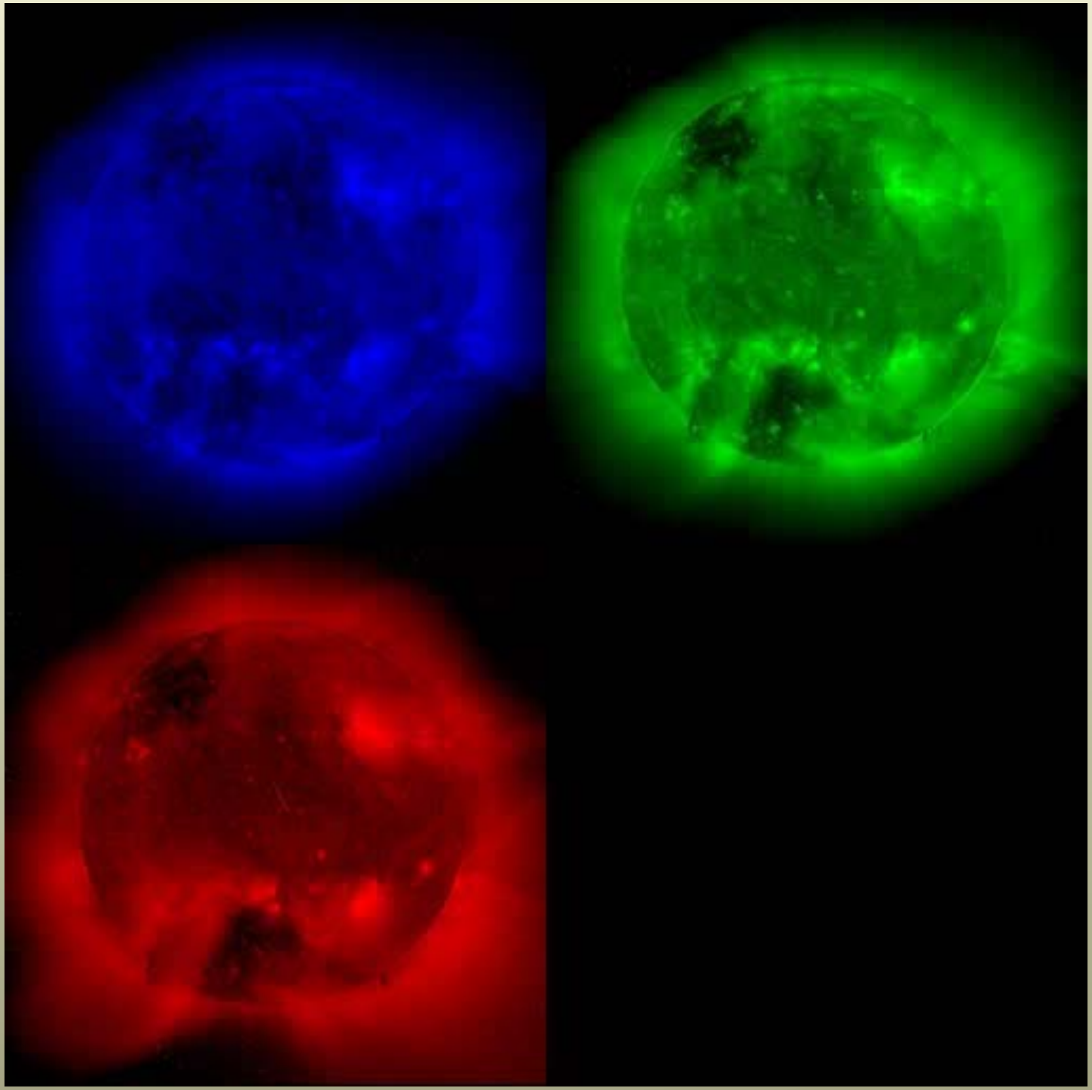
1. Basic Plasma Physics

- What is plasma?
- Where is plasma found?
- What consists of plasma?
- How do particles move in magnetic field?
- How does plasma evolve as fluid?
- What is plasma instability?
- What we need to know about plasma to understand radiation from moving particles?
- How do we investigate plasma dynamics?

Sunspots and activities observed by SOHO



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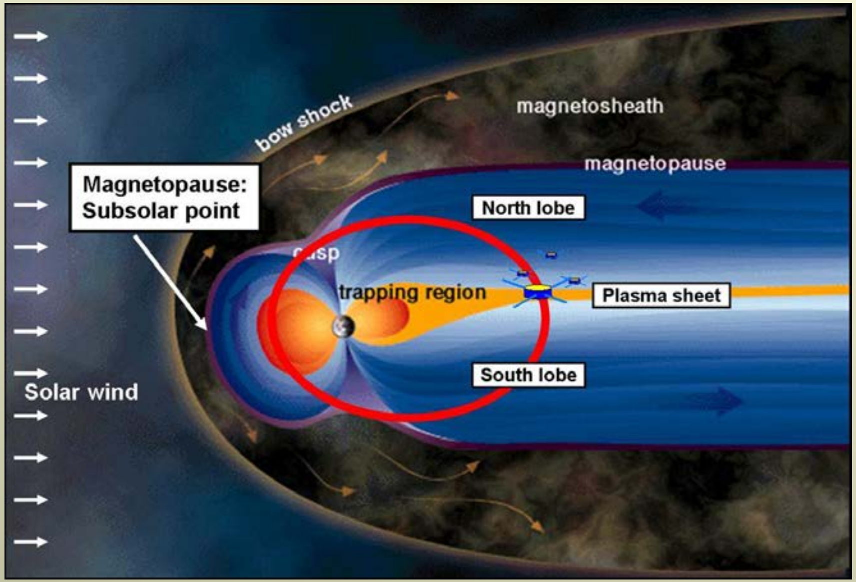
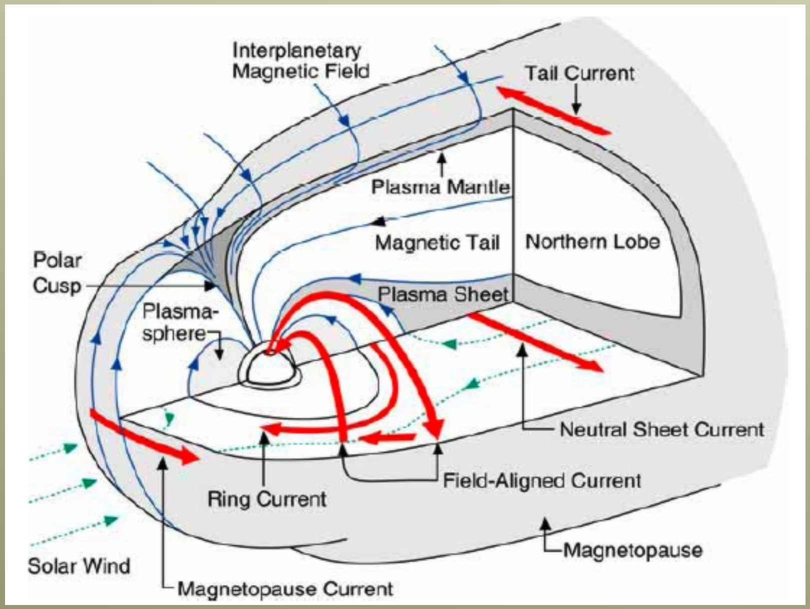
Three images (171A, 195A, and 284A) combined into one composite image

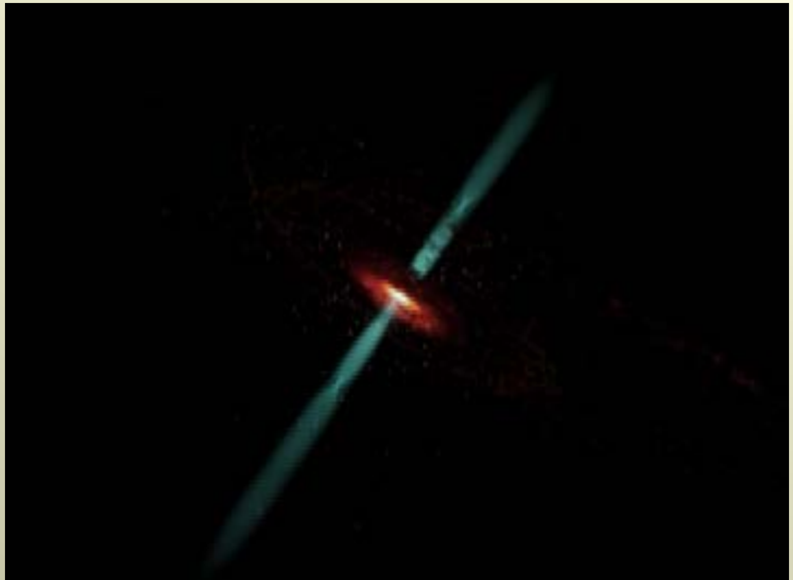
Solar wind-magnetosphere interaction: aurora

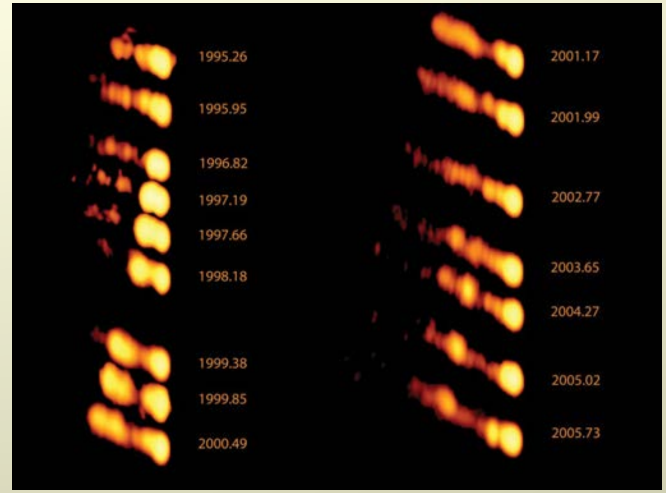


Geospace: Solar wind with interstellar magnetic fields (IMFs) interacts with Earth magnetosphere and create complicated structures and evolution: one of them is aurora

play yellowknife-combined.mov





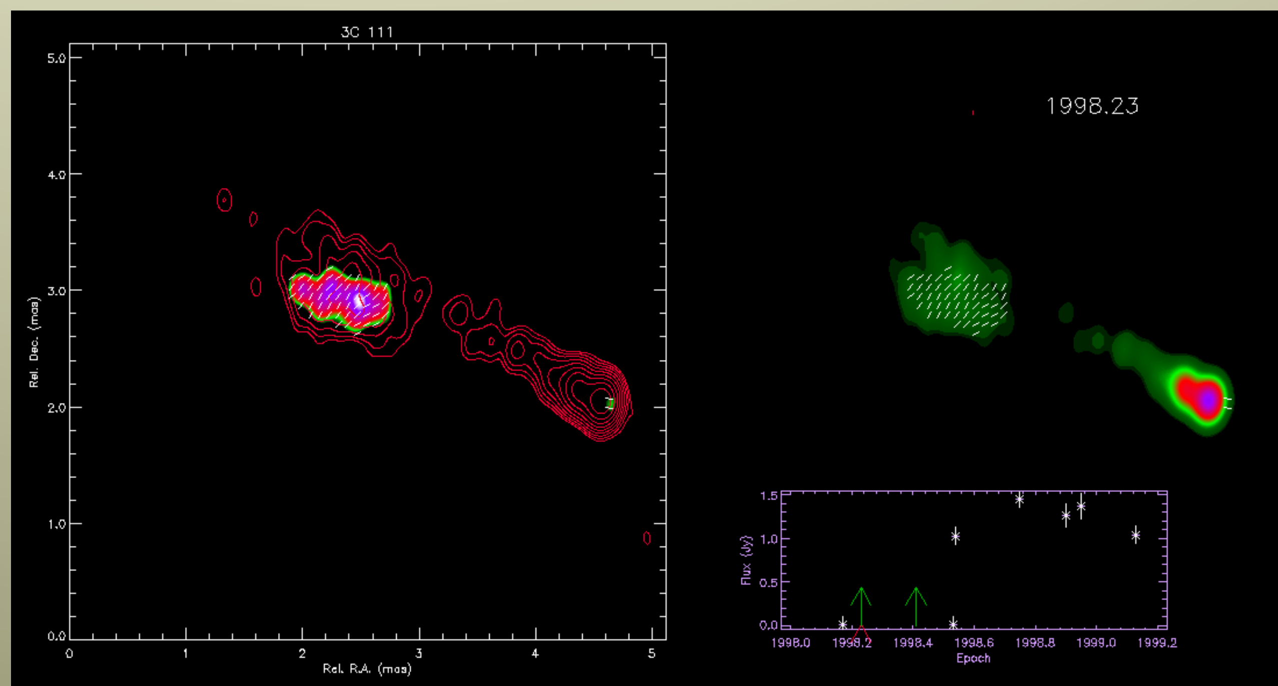


3C111 (Radio galaxy, z=0.0485)
observed by Very Long Baseline Array

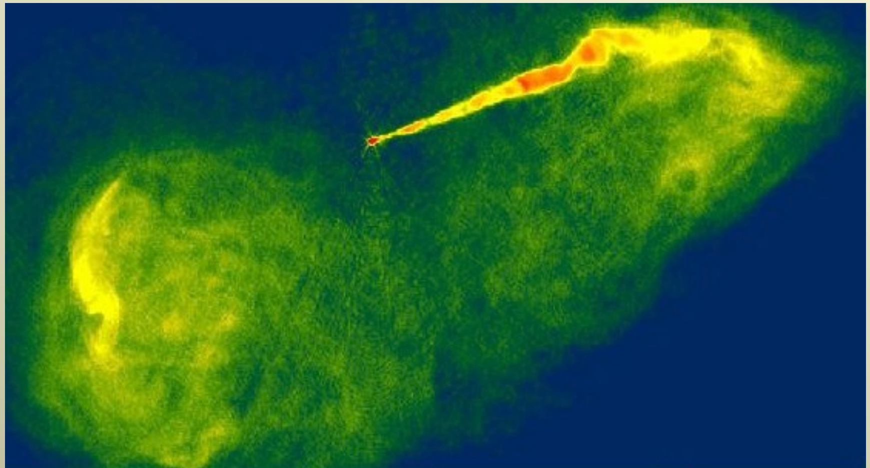
Black hole ejects
relativistic jets
play www-4.iaa.mp4

Relativistic jets

play 3c111vec.mov



High energy plasmas in astrophysics



Relativistic jets: M87

Supernova Remnants:
Cassiopeia A

Highly accelerated particles (electrons, positrons, and ions) are found in these astrophysical systems

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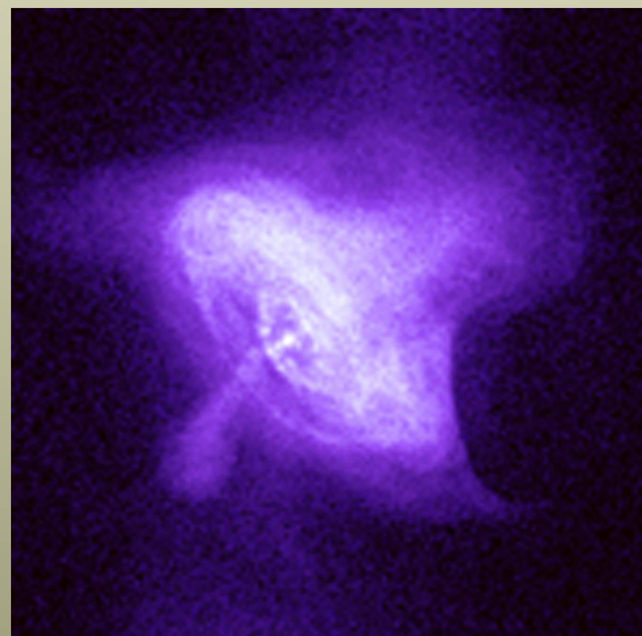


Pulsar Winds: Crab nebula

Particle acceleration occurs in shocks, reconnections, and other phenomena

What is plasma?

- Ionized gas:

  e^-: electrons, e^+: positrons, i^+: ions (p^+: protons)

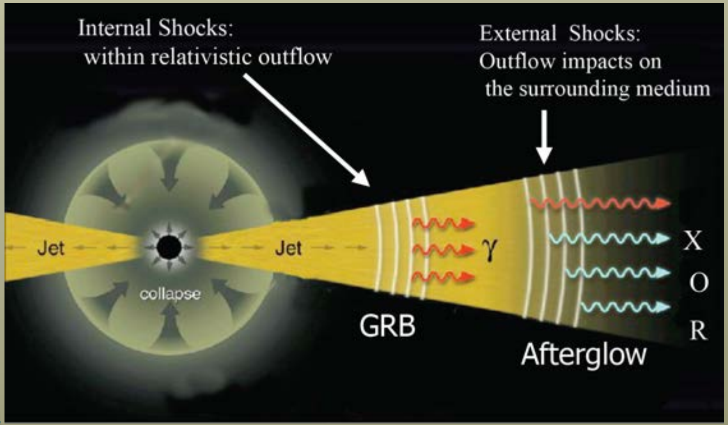
- Dairy examples: Florescent lamp (partially ionized)
- Controlled Fusion device (Tokamak, Laser Fusion, etc)
- Universe is consist of plasmas (99%)

  Active Galactic Nucleus (AGN) jets,

  Supernova Remnants (SNRs),

  Gamma-ray burst jets

  Neutron stars



Good text books for plasma physics

- Introduction to Plasma Physics and Controlled Fusion:

  vol. 1 Plasma Physics, by Frances F. Chen

- Principles of Plasma Physics,

  by N. A. Krall, and A. W. Trivelpiece

- Plasma Physics for Astrophysics,

  by Russell M. Kulsrud

- Plasma Physics Via Computer Simulation,

  by C.K. Birdsall, and A. B. Langdon

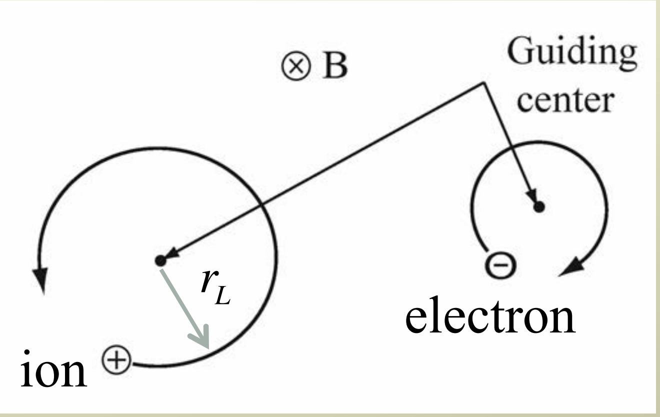
- Basic Space Plasma Physics and Advanced Space Plasma Physics

  by R. A. Treumann and W. Baumjohann

- many other good text books for instabilities, waves and nonlinear phenomena

Single particle motion

Uniform B and E



$$m\frac{dv}{dt} = q(E + v \times B)$$

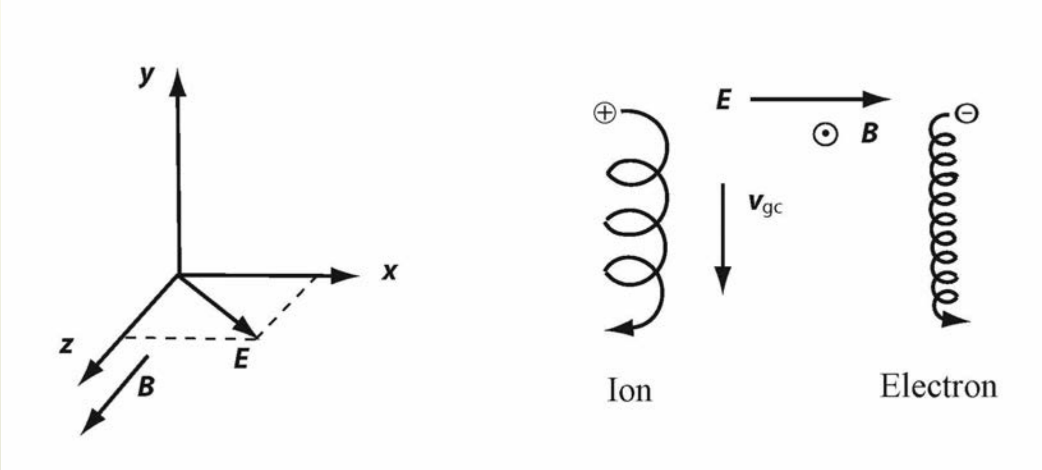
if $\boldsymbol{E} = 0$

$$m\dot{v}_x = qBv_y \qquad m\dot{v}_y = -qBv_x$$

$$\ddot{v}_x = \frac{qB}{m}\dot{v}_y = -\left(\frac{qB}{m}\right)^2 v_x \qquad \ddot{v}_y = -\frac{qB}{m}\dot{v}_x = -\left(\frac{qB}{m}\right)^2 v_y$$

Cyclotron frequency $\omega_c = \dfrac{|q|B}{m}$ Larmor radius $r_L \equiv \dfrac{v_\perp}{\omega_c} = \dfrac{mv_\perp}{|q|B}$

If E ≠0



z component

$$\frac{dv_z}{dt} = \frac{q}{m} E_z \qquad v_z = \frac{qE_z}{m} t + v_{t0}$$

transverse component

$$\frac{dv_y}{dt} = 0 \mp \omega_c v_x$$

$$\ddot{v}_x = -\omega_c^2 v_x \qquad \ddot{v}_y = -\omega_c^2 (v_y + \frac{E_x}{B})$$

$$v_x = v_\perp e^{i\omega_c t}$$

$$v_y = \pm i v_\perp e^{i\omega_c t} - \frac{E_x}{B}$$

if $\frac{dv}{dt} = 0$

$$E + v \times B = 0$$

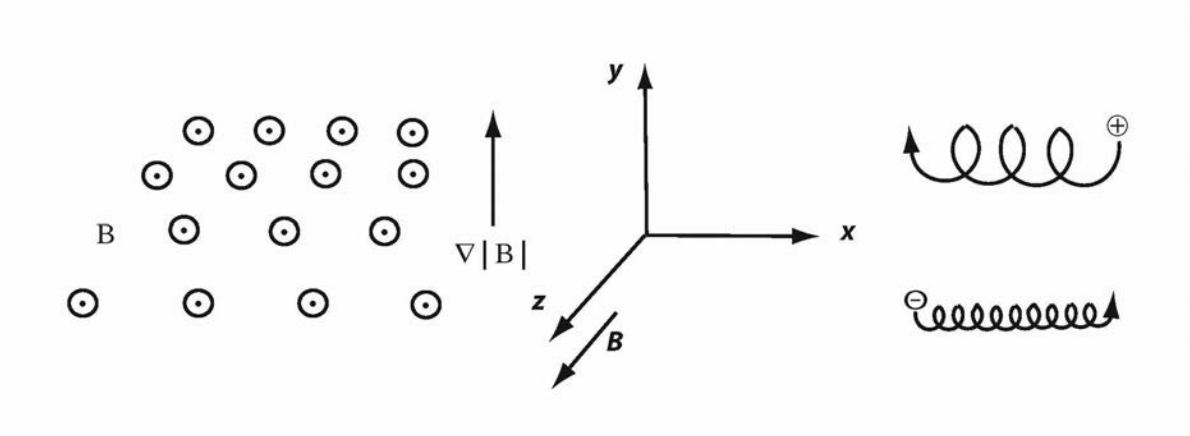
$$E \times B = B \times (v \times B) = vB^2 - B(v \bullet B)$$

E×B drift

$$\mathbf{v}_{\perp gc} = \mathbf{E} \times \mathbf{B} / B^2 \equiv \mathbf{v}_E$$

Nonuniform B field

$$\nabla B \perp \mathbf{B} : \text{Grad-}B$$



$$F_y = -qv_x B_z(y) = -qv_\perp(\cos\omega_c t)\left[B_0 \pm r_J(\cos\omega_c t)\frac{\delta B}{\delta y}\right] \longrightarrow \bar{F}_y = \mp q v_\perp r_L \frac{1}{2}\left(\frac{\partial B}{\partial y}\right)$$

$$v_{gc} = \frac{1}{q}\frac{\mathbf{F}\times\mathbf{B}}{B^2} = \frac{1}{q}\frac{\bar{F}_y}{|B|}\hat{\mathbf{x}} = \mp\frac{v_\perp r_L}{B}\frac{1}{2}\frac{\partial B}{\partial y}\hat{\mathbf{x}} \qquad v_{\nabla B} = \frac{1}{2}v_\perp r_L \frac{\mathbf{B}\times\nabla B}{B^2}$$

Summary of guiding center drifts

General force ***F***: $$\mathbf{v}_F = \frac{1}{q}\frac{\mathbf{F}\times\mathbf{B}}{B^2}$$

Electric field: $$\mathbf{v}_E = \frac{\mathbf{E}\times\mathbf{B}}{B^2}$$

Gravitational field: $$\mathbf{v}_g = \frac{m}{q}\frac{\mathbf{g}\times\mathbf{B}}{B^2}$$

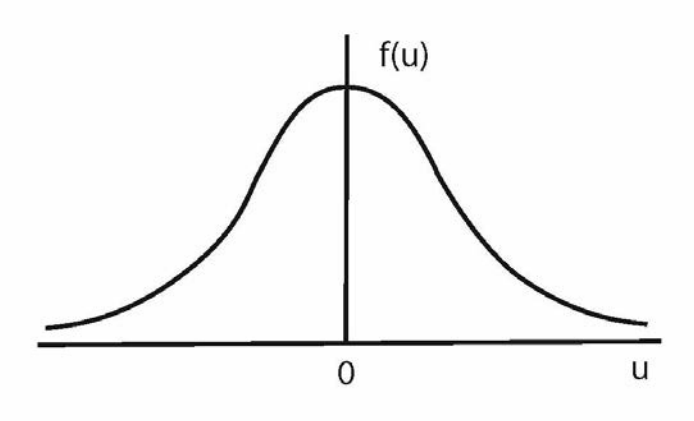
Nonuniform ***E*** : $$\mathbf{v}_E = \left(1+\frac{1}{4}r_L^2\nabla^2\right)\frac{\mathbf{E}\times\mathbf{B}}{B^2}$$

Grad-***B*** drift: $$\mathbf{v}_{\nabla B} = \pm\frac{1}{2}v_\perp r_L\frac{\mathbf{B}\times\nabla\mathbf{B}}{B^2}$$

Curved vacuum field: $$\mathbf{v}_R + v_{\nabla\mathbf{B}} = \frac{m}{q}\left(v_\parallel^2+\frac{1}{2}v_\perp^2\right)\frac{\mathbf{R}_c\times\mathbf{B}}{R_c^2B^2}$$

Polarization drift: $$\mathbf{v}_p = \pm\frac{1}{\omega_c B}\frac{d\mathbf{E}}{dt}$$

Concept of temperature



One-dimensional Maxwellian distribution

$$f(u) = A\exp(-\frac{1}{2}mu^2/KT)$$

K: Boltzman's constant $K = 1.38\times 10^{-23}\,\mathrm{J/\,°K}$

$KT = 1\mathrm{eV} = 1.6\times 10^{-19}\,\mathrm{J}$

Density n/m^2 $\quad n = \int_{-\infty}^{\infty} f(u)\,du$

$1\mathrm{eV} = 11{,}600\,°\mathrm{K}$

$$E_{av} = \frac{\int_{-\infty}^{\infty}\frac{1}{2}mu^2 f(u)\,du}{\int_{-\infty}^{\infty} f(u)\,du}$$

$$v_{th} = (2KT/m)^{1/2}$$

$$E_{av} = \frac{1}{4}mv_{th}^2 = \frac{1}{2}KT$$

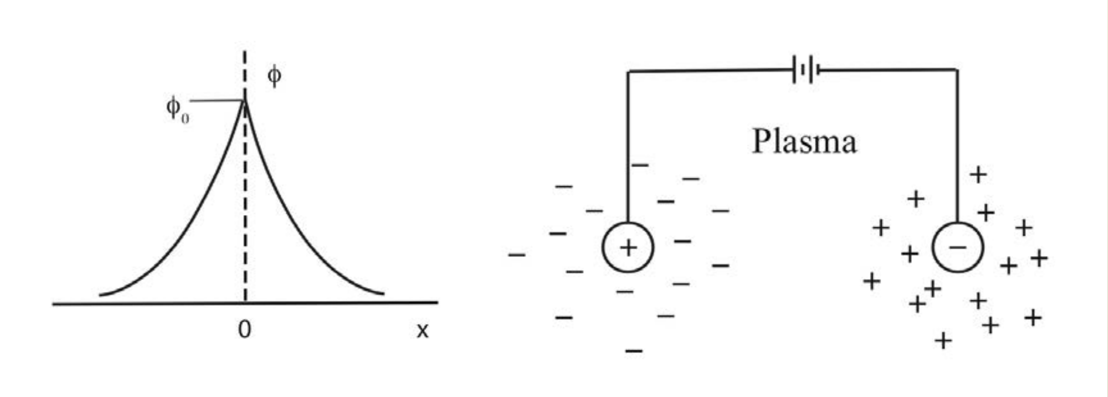
Three-dimensional Maxwell's distribution

$$f(u,v,w) = A_3\exp[-\frac{1}{2}m(u^2+v^2+w^2)/KT]$$

$$A_3 = n\left(\frac{m}{2\pi KT}\right)^{3/2}$$

$$E_{av} = \frac{3}{2}KT$$

Debye shielding



Poisson's equation

$$\varepsilon_0 \nabla^2 \phi = \varepsilon_0 \frac{d^2\phi}{dx^2} = -(n_i - n_e) \qquad n_i = n_\infty$$

Electron distribution function

$$f(u) = A \exp\left[-(\frac{1}{2} m u^2 + q\phi)/KT \right] \qquad n_e = n_\infty \exp(e\phi/KT_e)$$

$$\varepsilon_0 \frac{d^2\phi}{dx^2} = e n_\infty \left\{ \left[\exp\left(\frac{e\phi}{KT_e} \right) \right] - 1 \right\} \approx e n_\infty \left[\frac{e\phi}{KT_e} + \frac{1}{2} \left(\frac{e\phi}{KT_e} \right)^2 + \cdots \right]$$

$$\varepsilon_0 \frac{d^2\phi}{dx^2} = \frac{n_\infty e^2}{KT_e} \phi \qquad \phi = \phi_0 \exp(-|x| / \lambda_D)$$

Debye length

$$\lambda_D = \left(\frac{\varepsilon_0 K T_e}{n_e^2} \right)^{½}$$

Maxwell equations

In a medium

$$\nabla \bullet \mathbf{D} = \sigma \qquad\qquad \mathbf{D} = \varepsilon \mathbf{E}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \bullet \mathbf{B} = 0$$

$$\nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t} \qquad\qquad \mathbf{H} = \mathbf{B}/\mu$$

Fluid equation of motion

$$mn\left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \bullet \nabla)\mathbf{u}\right] = qn(\mathbf{E} + \mathbf{u} \times \mathbf{B}) - \nabla \bullet \mathbf{P} - \frac{mn(\mathbf{u} - \mathbf{u}_0)}{\tau}$$

Complete set of fluid equations

$$\varepsilon_0 \nabla \bullet \mathbf{E} = \sigma = n_i q_i + n_e q_e$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \bullet \mathbf{B} = 0$$

$$\mu_0^{-1} \nabla \times \mathbf{B} = \mathbf{j} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} = n_i q_i \mathbf{v}_i + n_e q_e \mathbf{v}_e + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$m_j n_j \left[\frac{\partial \mathbf{v}_j}{\partial t} + (\mathbf{v}_j \bullet \nabla) \mathbf{v}_j \right] = q_j n_j (\mathbf{E} + \mathbf{v}_j \times \mathbf{B}) - \nabla \bullet \mathbf{P}_j \qquad j = i, e$$

Plasma oscillations (1-dimensional)

Electron equations of motion and continuity

$$n_e = n_0 + n_1 \qquad \mathbf{v}_e = \mathbf{v}_0 + \mathbf{v}_1 \qquad \mathbf{E} = \mathbf{E}_0 + \mathbf{E}_1$$

Basic equations

$$mn_e\left[\frac{\partial \mathbf{v}_e}{\partial t} + (\mathbf{v}_e \bullet \nabla)\mathbf{v}_e)\right] = -en_e\mathbf{E}$$

$$\frac{\partial n_e}{\partial t} + \nabla \bullet (n_e \mathbf{v}_e) = 0$$

$$\varepsilon_0 \nabla \bullet \mathbf{E} = \varepsilon_0 \partial \mathbf{E} / \partial \mathbf{x} = e(n_i - n_e)$$

Linear analysis

$$m\left[\frac{\partial \mathbf{v}_1}{\partial t} + (\mathbf{v}_1 \bullet \nabla)\mathbf{v}_1\right] = -e\mathbf{E}_1$$

$$\frac{\partial n_1}{\partial t} + \nabla \bullet (n_0 \mathbf{v}_1 + n_1 \mathbf{v}_1) = \frac{\partial n_1}{\partial t} + n_0 \nabla \bullet \mathbf{v}_1 = 0$$

$$\varepsilon_0 \nabla \bullet \mathbf{E}_1 = \varepsilon_0 \partial \mathbf{E}_1 / \partial \mathbf{x} = -en_1$$

Plasma oscillations (1-dimensional)

Assumed perturbations (sinusoildal)

$$\mathbf{v}_1 = v_1 e^{i(kx-\omega t)} \widehat{\mathbf{x}} \qquad n_1 = n_1 e^{i(kx-\omega t)} \qquad \mathbf{E} = E_1 e^{i(kx-\omega t)} \widehat{\mathbf{x}}$$

Linearlization of equations

$$-im\omega v_1 = -eE_1 \qquad -i\omega n_1 = -n_0 ikv_1 \qquad -ik\varepsilon_0 E_1 = -en_1$$

Eliminating n_1 and E_1

$$-im\omega v_1 = -e\frac{-e}{ik\varepsilon_0}\frac{-n_0 ikv_1}{-i\omega} = i\frac{n_0 e^2}{\varepsilon_0 \omega} v_1 \qquad \omega^2 = n_0 e^2 / m\varepsilon_0$$

Plasma frequency $\omega_p = \left(\frac{n_0 e^2}{m\varepsilon_0}\right)^{1/2}$ $\qquad f_p = \omega_p/2\pi \approx 9\sqrt{n}$

$$n = 10^{18}\,\mathrm{m}^{-3} \qquad B \approx 0.32\mathrm{T}{=}0.32\times 10^4\mathrm{G}\ \text{(Gauss)}$$

$$f_p \approx 9(10^{18})^{1/2} = 9\times 10^9\,\sec^{-1} = 9\mathrm{GHz} \simeq \mathrm{f}_{ce} \simeq 28\mathrm{GHz/Tesla}$$

Single-fluid equations

Quasineutral plasma with singly charged ions

$$Mn\frac{\partial \mathbf{v}_i}{\partial t} = en(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) - \nabla p_i + Mn\mathbf{g} + \mathbf{P}_{ie}$$

$$mn\frac{\partial \mathbf{v}_e}{\partial t} = -en(\mathbf{E} + \mathbf{v}_e \times \mathbf{B}) - \nabla p_e + mn\mathbf{g} + \mathbf{P}_{ei}$$

$$Mmn\frac{\partial}{\partial t}(\mathbf{v}_i - \mathbf{v}_e) = en(M+m)\mathbf{E} + en(m\mathbf{v}_i - M\mathbf{v}_e)\times\mathbf{B} - m\nabla p_i + M\nabla p_e - (M+m)\mathbf{P}_{ei}$$

$$\rho \equiv n_i M + n_e m \approx n(M+m) \qquad p = p_i + p_e$$

$$\mathbf{v} \equiv \frac{1}{\rho}\left(n_i M\mathbf{v}_i + n_e m\mathbf{v}_e\right) \approx \frac{M\mathbf{v}_i + m\mathbf{v}_e}{M+m}$$

$$\mathbf{j} \equiv e\left(n_i\mathbf{v}_i - n_e\mathbf{v}_e\right) \approx ne(\mathbf{v}_i - \mathbf{v}_e)$$

$$\sigma = n_i q_i + n_e q_e$$

Single-fluid equations

$$\mathbf{E}+\mathbf{v}\times\mathbf{B}=\eta\mathbf{j}+\frac{1}{en}(\mathbf{j}\times\mathbf{B}-\nabla p_e) \qquad \eta=\frac{m}{ne^2}\nu_{ei}=\frac{e^2}{16\pi\varepsilon_0^2 mv^2}\approx\frac{\pi e^2 m^{1/2}}{(4\pi\varepsilon_0)^2(KT_e)^{3/3}}\ln\Lambda \qquad \Lambda=\overline{\lambda_D/r_0}$$

$$\frac{Mmn}{e}\frac{\partial}{\partial t}\left(\frac{\mathbf{j}}{n}\right)=e\rho\mathbf{E}-(M+m)ne\eta\mathbf{j}-m\nabla p_i+M\nabla p_e+en(m\mathbf{v}_i+M\mathbf{v}_e)\times\mathbf{B}$$

$$m\mathbf{v}_i+M\mathbf{v}_e=M\mathbf{v}_i+m\mathbf{v}_e+M(\mathbf{v}_e-\mathbf{v}_i)+m(\mathbf{v}_i-\mathbf{v}_e)=\frac{\rho}{n}\mathbf{v}-(M-m)\frac{\mathbf{j}}{ne}$$

Generalized Ohm's law

$$\mathbf{E}+\mathbf{v}\times\mathbf{B}-\eta\mathbf{j}=\frac{1}{e\rho}\left[\frac{Mmn}{e}\frac{\partial}{\partial t}\left(\frac{\mathbf{j}}{n}\right)+(M-m)\mathbf{j}\times\mathbf{B}+m\nabla p_i-M\nabla p_e\right]$$

$$\mathbf{E}+\mathbf{v}\times\mathbf{B}=\eta\mathbf{j}+\frac{1}{en}(\mathbf{j}\times\mathbf{B}-\nabla p_e)$$

Set of MHD equations

equation of motion

$$\rho \frac{\partial \mathbf{v}}{\partial t} = \mathbf{j} \times \mathbf{B} - \nabla p + \rho \mathbf{g}$$

simplified Ohm's law

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{j}$$

equation of continuity

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \mathbf{v}) = 0$$

conservation of charge

$$\frac{\partial \sigma}{\partial t} + \nabla \bullet \mathbf{j} = 0$$

Kinetic theory

For some phenomena fluid theory is inadequate

number of particles

$$f(x, y, z, v_x, v_y, v_z, t)\,dv_x dv_y dv_z$$

$$n(\mathbf{r},t) = \int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} dv_z f(\mathbf{r},\mathbf{v},t) = \int_{-\infty}^{\infty} f(\mathbf{r},\mathbf{v},t)\,d^3v = \int_{-\infty}^{\infty} f(\mathbf{r},\mathbf{v},t)\,d\mathbf{v}$$

$$f(\mathbf{r},\mathbf{v},t) = n(\mathbf{r},t)\hat{f}(\mathbf{r},\mathbf{v},t) \qquad \hat{f}_m = (m/2\pi KT)^{3/2}\exp(-v^2/v_{th})$$

$$v = (v_x^2 + v_y^2 + v_z^2)^{1/2} \qquad v_{th} = (2KT/m)^{1/2}$$

Vlasov equation

$$\frac{\partial f}{\partial t} + \mathbf{v} \bullet \nabla \mathbf{f} + \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \bullet \frac{\partial f}{\partial \mathbf{v}} = 0$$

Fokker-Planck equation

$$\frac{\partial f}{\partial t} + \mathbf{v} \bullet \nabla \mathbf{f} + \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \bullet \frac{\partial f}{\partial \mathbf{v}} = \left(\frac{\partial f}{\partial t}\right)_c$$

Summary

Plasma

Kinetic picture

Particle motion
acceleration
radiation

Fluid picture

Global dynamics
Plasma temperature

Linear theory

Waves, instability

Nonlinear theory

Linear theory breaks down, saturation

Particle-in-Cell simulations

Local microscopic phenomena

MHD simulations

Global system