Lecture 5:

Application of General Relativistic Magnetohydrodynamics

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Spacial lecture “GRMHD”, August 13th-17th, USP-IAG, Sao Paulo, Brazil

Applications of Relativistic Astrophysics

- Black Holes:
  - high, low accretion rate AGN
  - tidal disruption event
  - X-ray binaries
  - long-soft GRBs
  - BH-BH merger for GW sources

- Neutron stars:
  - pulsar magnetosphere
  - core-collapse supernova
  - short-hard GRBs
  - NS-NS merger for GW sources

- Jets/relativistic wind:
  - extra-galactic jets/outflows
  - pulsar jet/wind
  - microquasars
  - gamma-ray bursts

- Laboratory physics:
  - relativistic heavy-ion collision
  - plasma laboratory experiments

Standard picture: plasma accretion onto a black hole

Modes of Accretion

- Low accretion rate

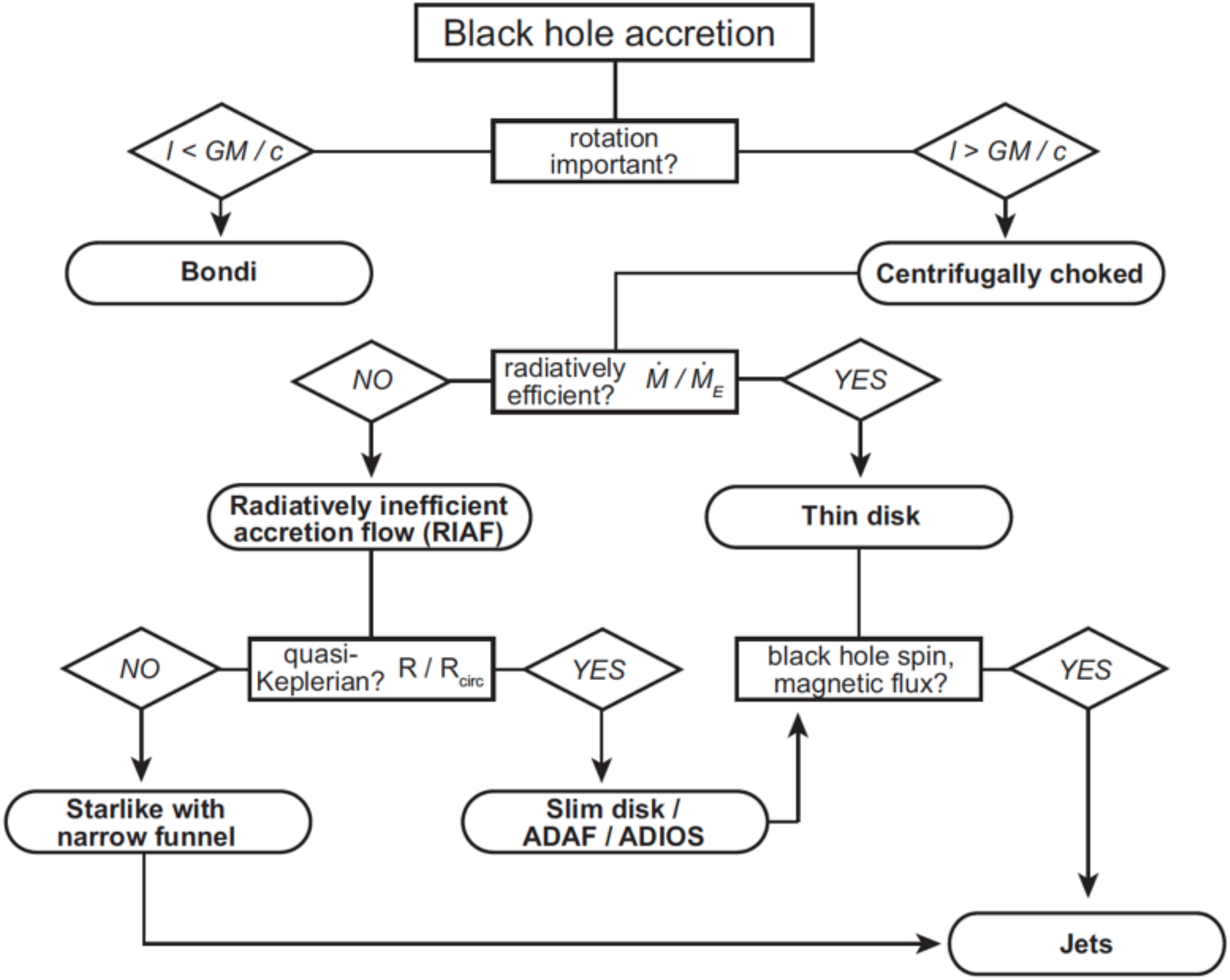
  Radiatively Inefficient Accretion Flow (RIAF)
  optically thin, geometrically thick

- High accretion rate

  classical standard disk (Shakura-Sunyaev)
  optically thick, geometrically thin

Black Hole Accretion

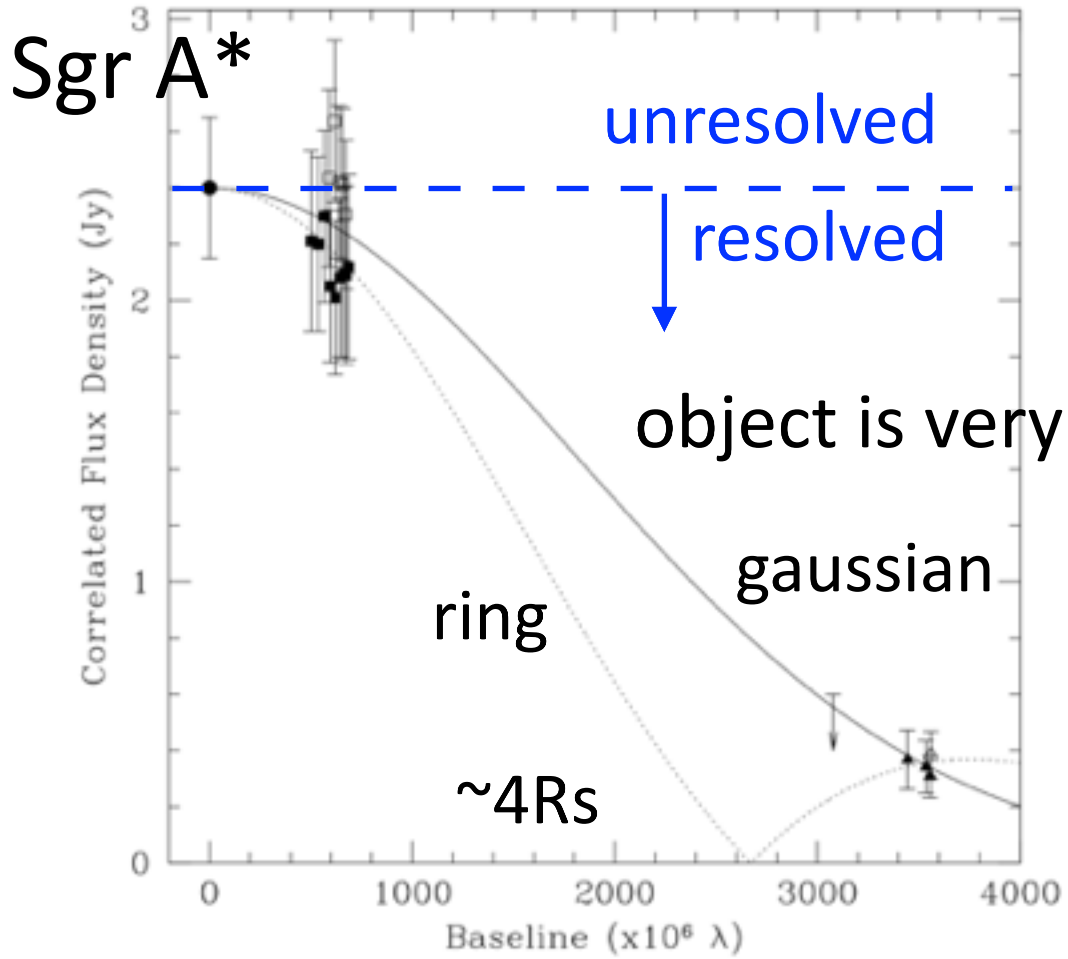
Begelman (2014)



Event Horizon Telescope (EHT): VLBI Images of Black Holes

- Two largest Black Holes in the sky
  - Sgr A* and M87, both low-luminous AGNs

Early EHT observation: CALMA-SMT-SMA/JCMT



Sgr A*

unresolved

resolved

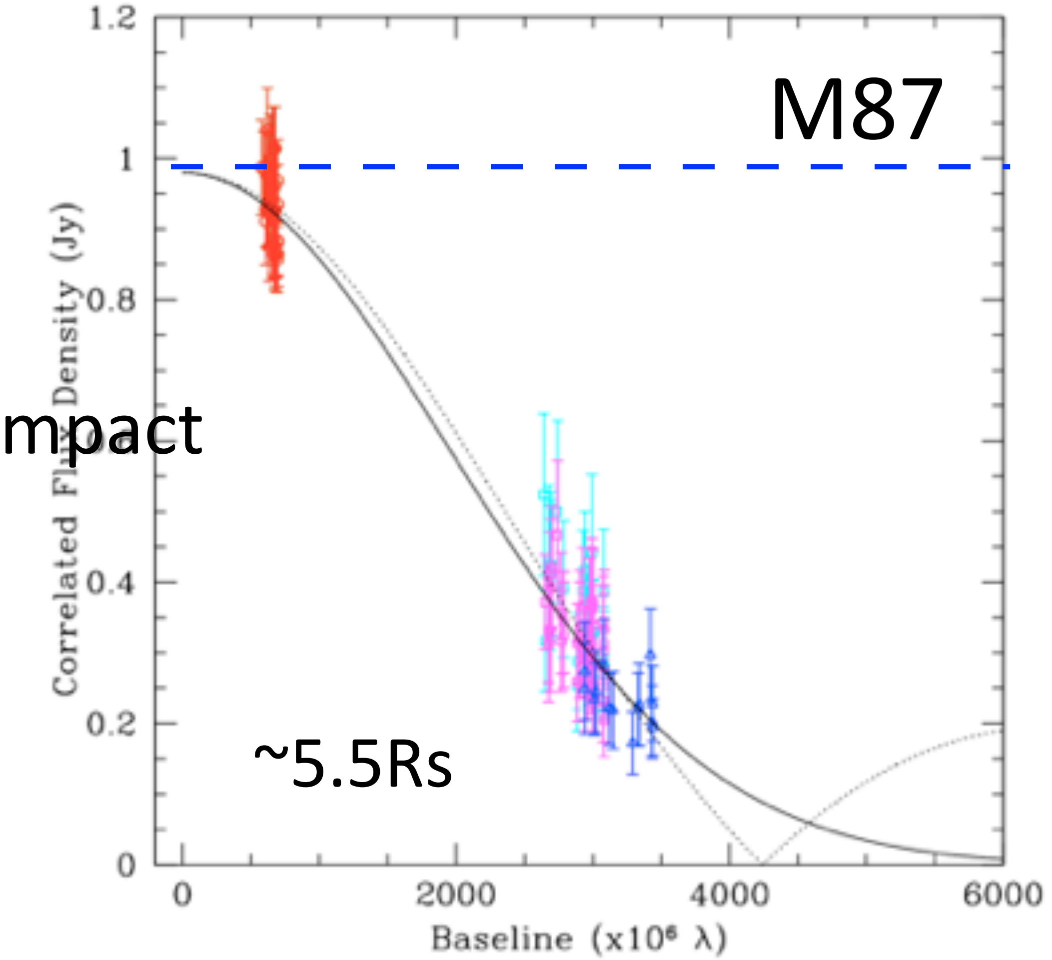
object is very compact

ring

gaussian

~4Rs

Doeleman et al. (2008)

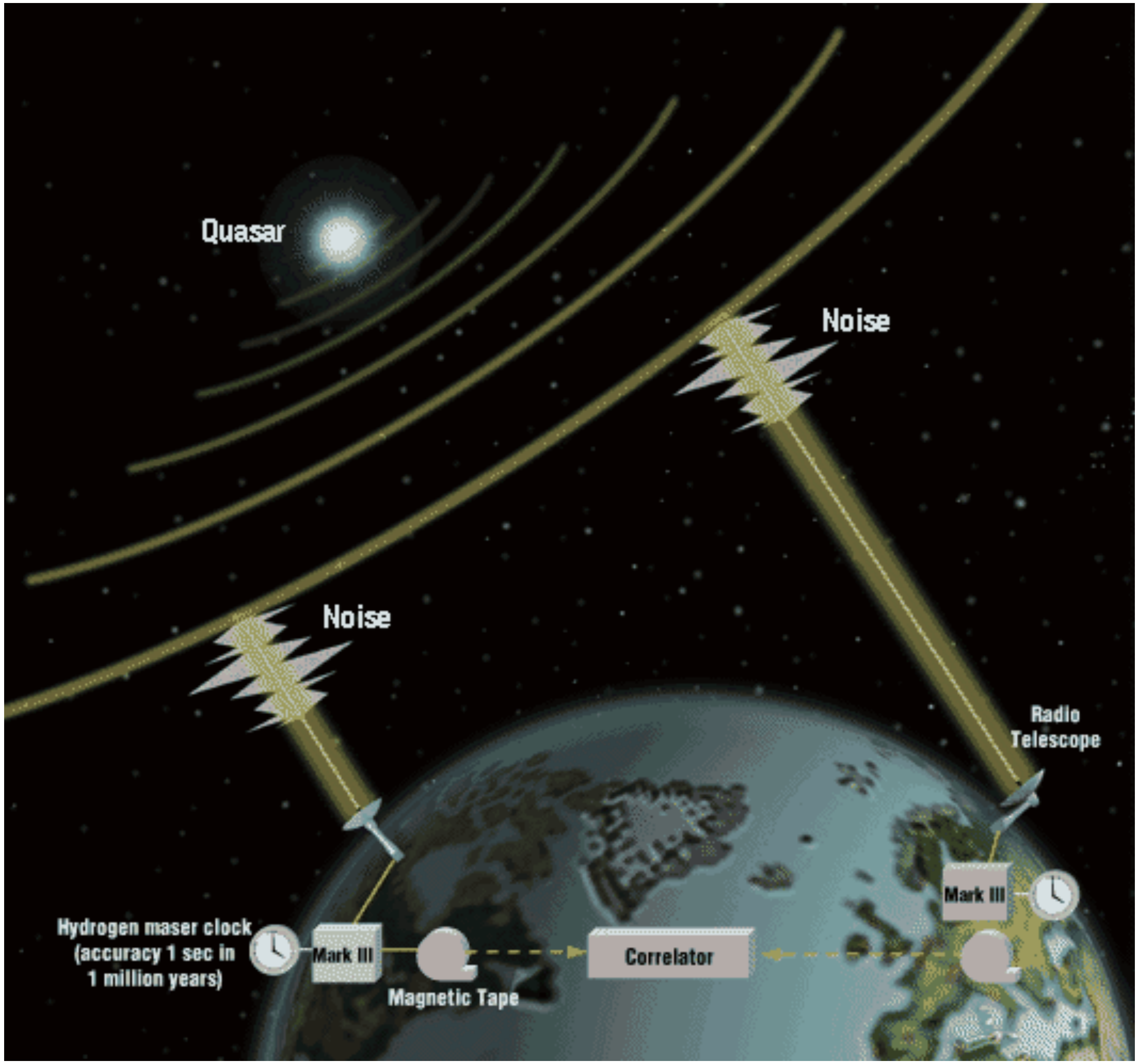


M87

~5.5Rs

Doeleman et al. (2012)

Short Wavelength VLBI



Angular Resolution:

λ/D (cm) ~ 0.5 mas
λ/D (1.3mm) ~ 30 μas
λ/D (0.8mm) ~ 20 μas

ISM scatter (Sgr A*):

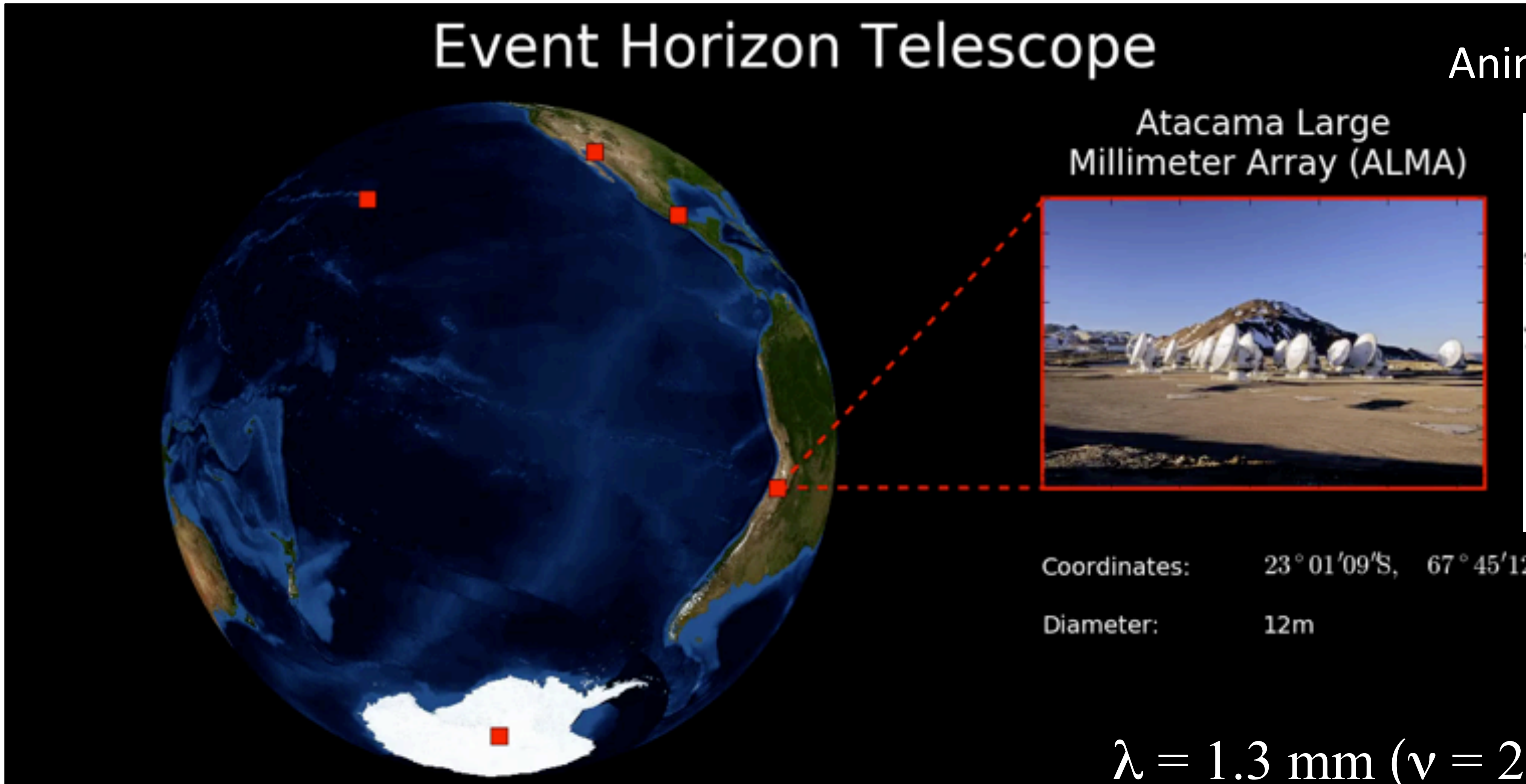
$\Theta_{scat} \sim \lambda^2$

BH Shadow size:

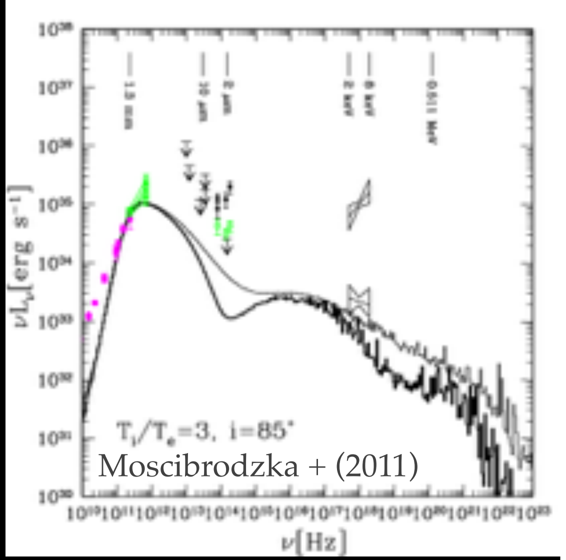
Sgr A*: 50 μas
M87: 40 μas

Event Horizon Telescope

International collaboration project of Very Long Baseline Interferometry (VLBI) at mm (sub-mm) wavelength



Animation: C. Fromm



Moscibrodzka + (2011)

Create a virtual radio telescope the size of the earth, using the shortest wavelength

$\lambda = 1.3$ mm ($\nu = 230$ GHz)
D ~ 10,000 km
=> $\lambda/D \sim 25$ μas

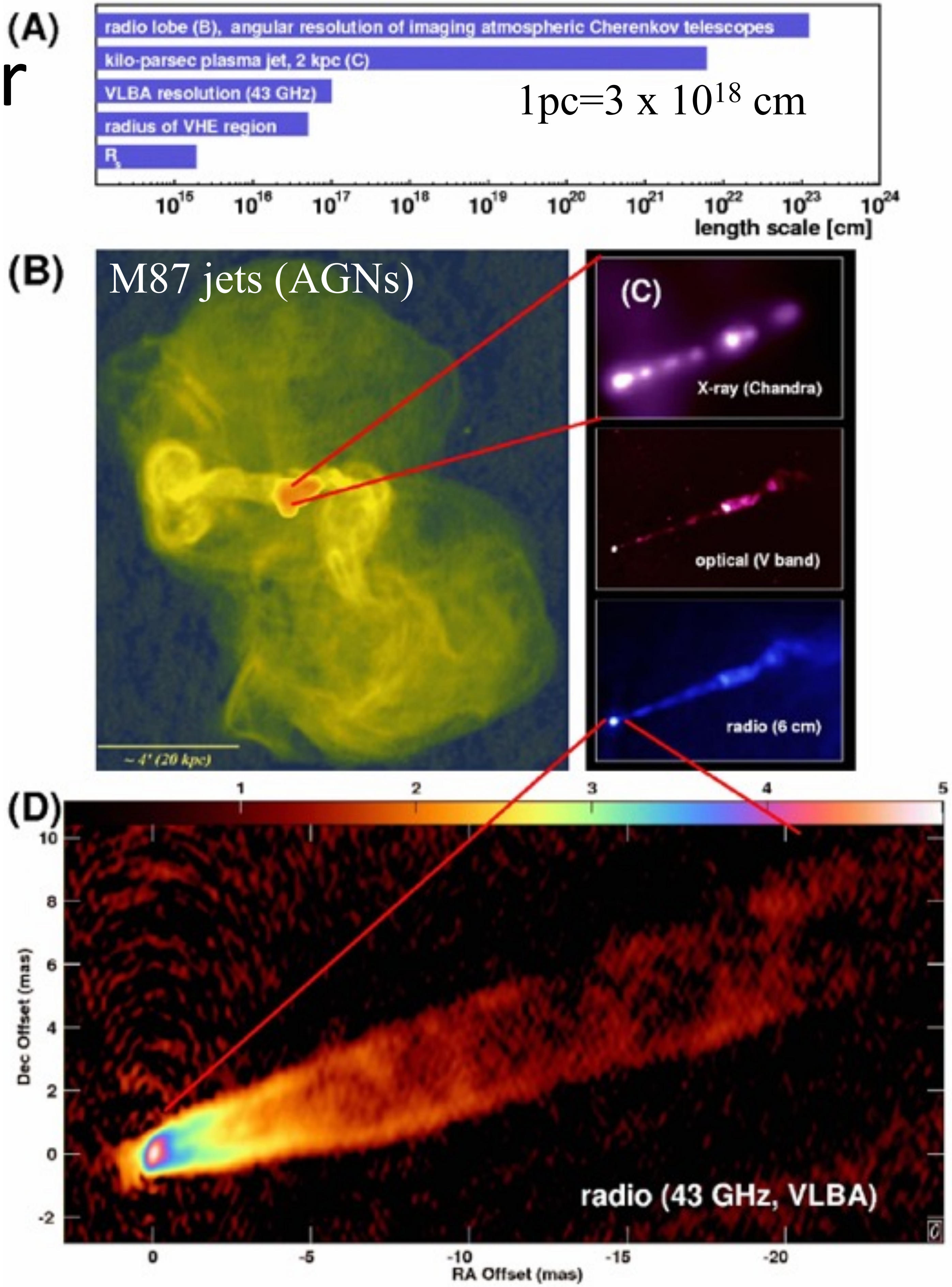
Two main targets: Sgr A* & M87

Sgr A* vs M87

	M87	Sgr A*
Mass (M_{sun})	3-6 x 10^9 (?)	4 x 10^6
Distance	16 Mpc	8.5 kpc
Luminosity	10^{44} erg/s	10^{36} erg/s
Mdot (M_{edd})	10^{-4}	10^{-8}
BH Spin Axis	Gal disk?	10-25 deg los
@ the BH?	Maybe	Yes
B field @ BH	60-130 G	10-100 G
Scattered?	No	yes
Shadow Size	640 AU	0.5 AU
Shadow Angle	20-40 μas	52 μas
GM/c3	8 hrs	20 sec
ISCO Period	4-54 days	4-54 min
Jet Power	10^{42}-10^{43} erg/s	?

M87 is the best object for relativistic jet study

- Relativistic jet is a tremendous, elongated and collimated outflows of plasma with relativistic speed
- Lunching from accreting compact objects (Black Holes)
- M87 is observed huge spatial range from Mpc to < 1pc.
- Observed large-scale relativistic jet is kinetic energy is dominated



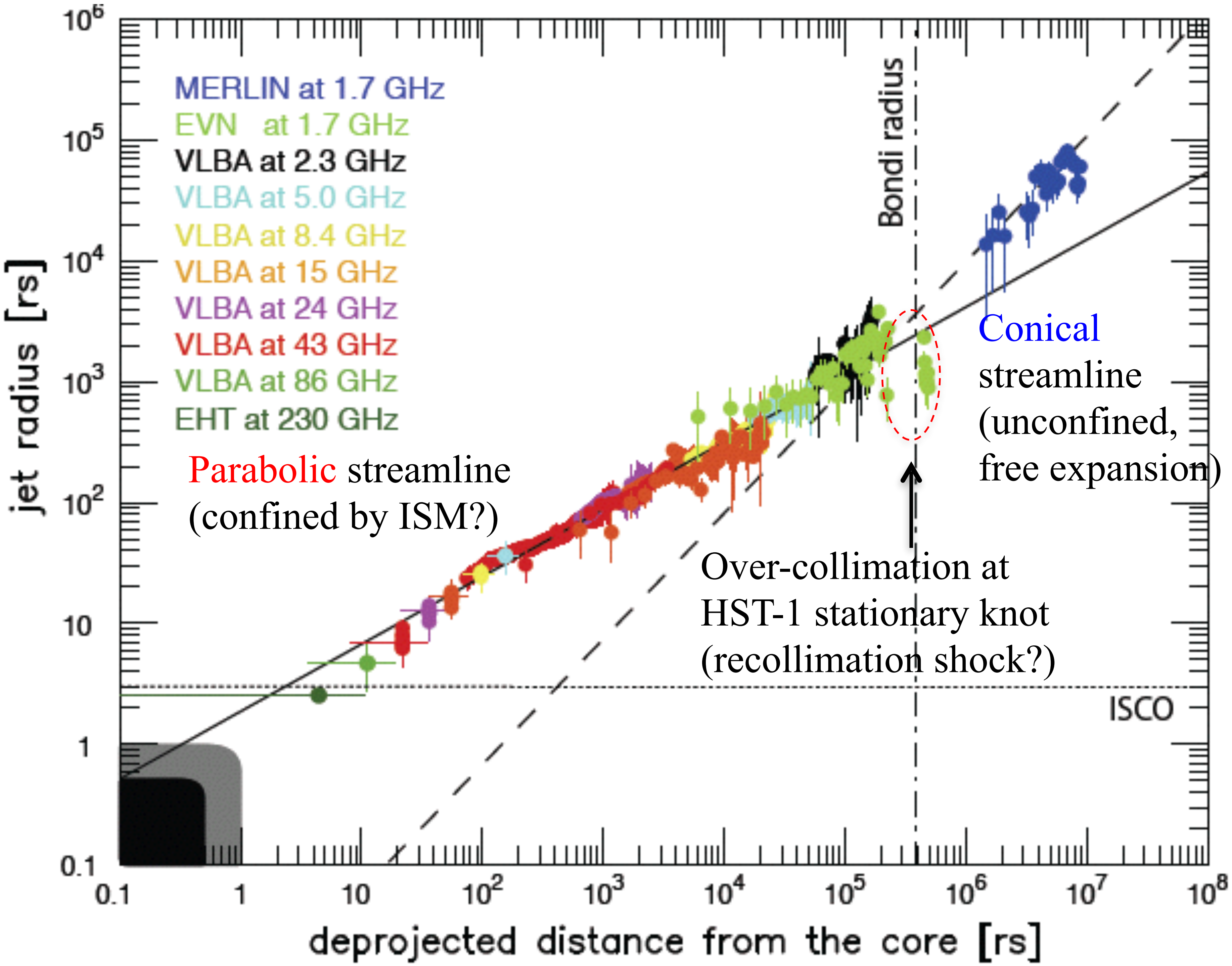
1pc=3 x 10^{18} cm

M87 jets (AGNs)

Global structure of M87 jet

Asada & Nakamura (2012),
Hada et al. (2013)

- The parabolic structure ($z \propto r^{1.7}$) maintains over $10^5\ r_s$, external confinement is worked.
- The transition of streamlines presumably occurs beyond the gravitational influence of the SMBH (= Bondi radius)
- In far region, jet stream line is conical ($z \propto r$)
- Stationary feature HST-1 is a consequence of the jet recollimation due to the pressure imbalance at the transition

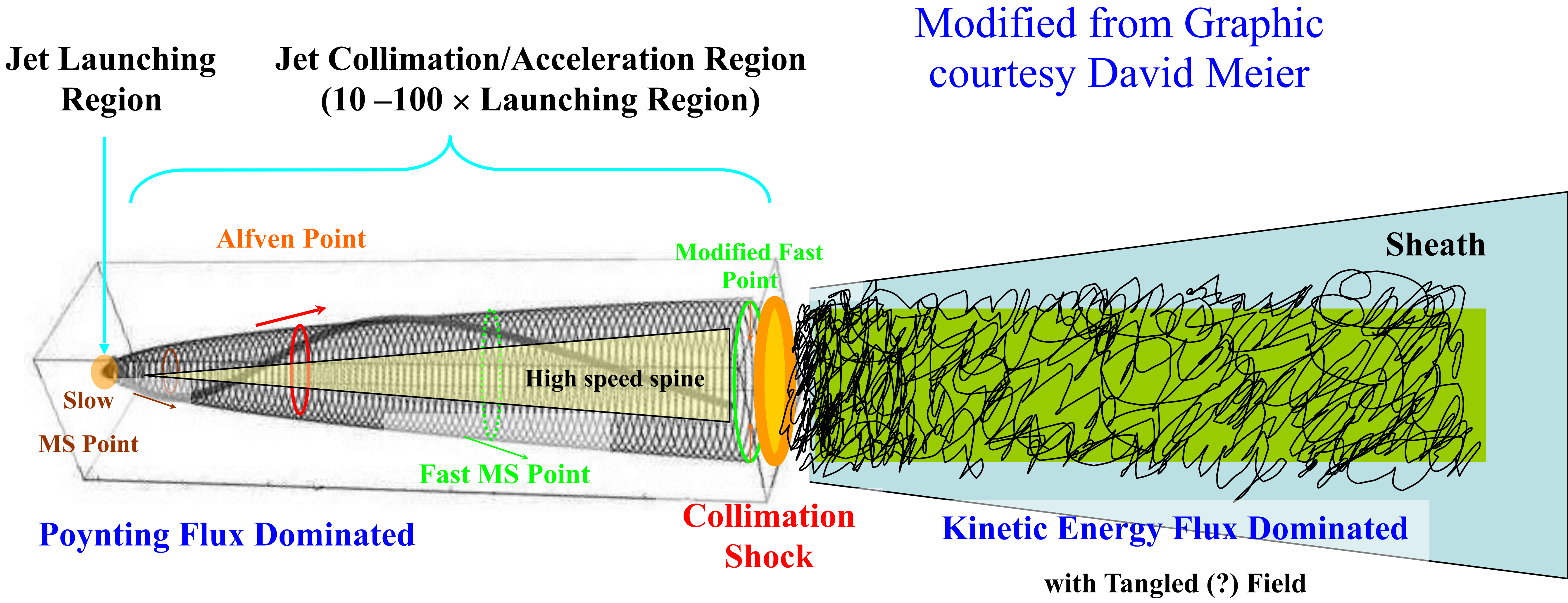


Parabolic streamline (confined by ISM?)

Conical streamline (unconfined, free expansion)

Over-collimation at HST-1 stationary knot (recollimation shock?)

Regions of AGN Jet Propagation



- Jet launching by MHD process => Poynting flux dominated jet with twisted magnetic field
- Need rapid magnetic energy dissipation to make a kinetic energy dominated jet

Jet formation/acceleration mechanism

- Jet is formed near the central compact objects (BHs/NSs).
- Some accreting matter is getting some force to make jet-like outflows.
- Ingredients: rotation, accretion disk, magnetic fields
- Jet base: rotating disk or compact objects (BHs/NSs)
- The jet formation/acceleration mechanism is still under debate but …
- The most promising mechanism is the acceleration/formation by rotating, twisting magnetic fields (magnetohydrodynamic (MHD) process)
- Other possibility: gas pressure, radiation pressure, …

Jet formation/acceleration mechanism

- Gas or radiation pressure (Blandford & Rees 1974, O'Dell 1984)
  - push accretion matter to make and accelerate outflows by pressure gradient

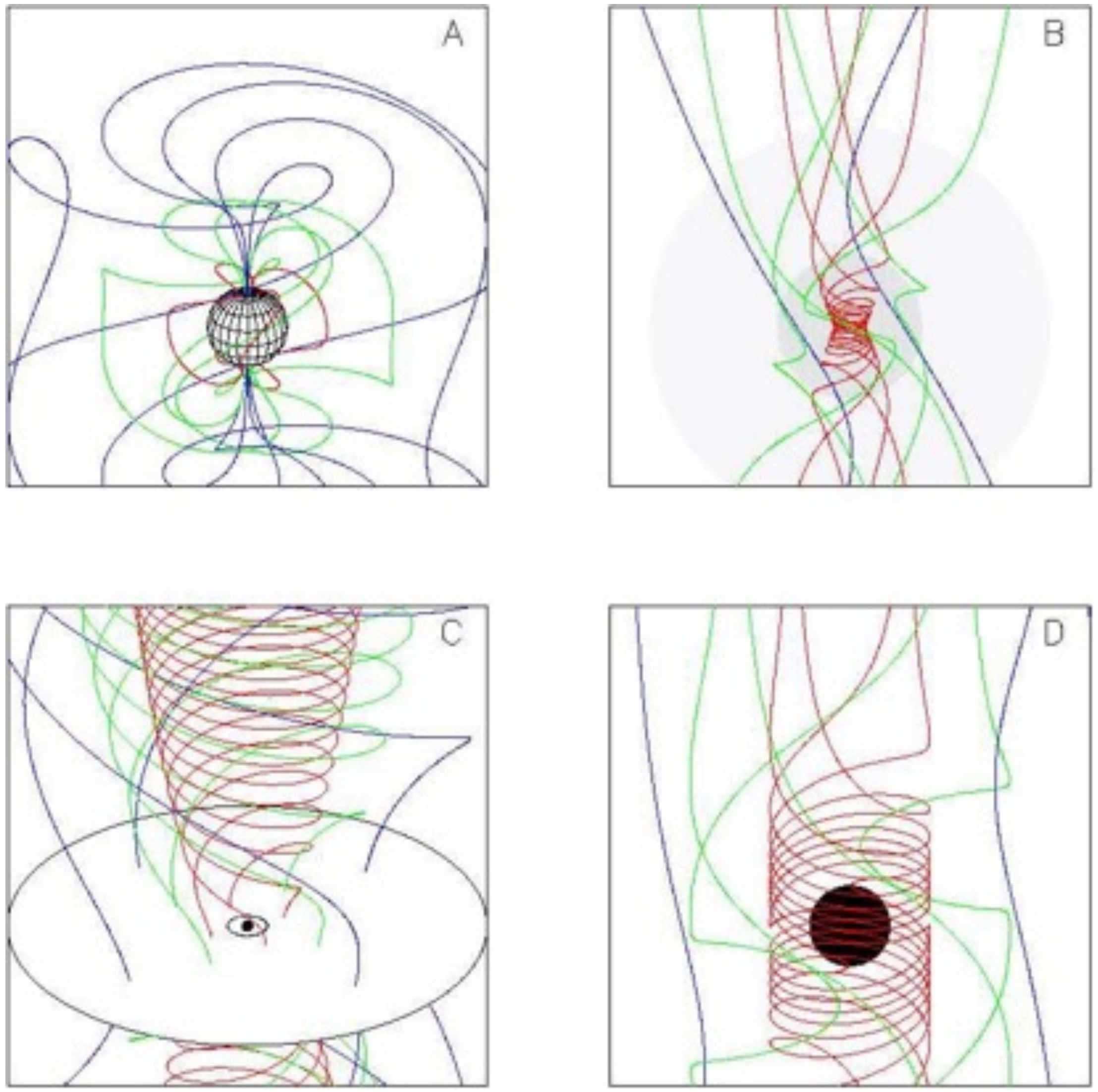
- Expansion of magnetic tower (Lynden-Bell & Boily 1994)
  - Mainly toroidal field from start
  - Acceleration by magnetic pressure

- Magnetocentrifugal acceleration (Blandford & Payne 1982)
  - Mainly poloidal field anchored to disk or rotating objects
  - Disk or ergosphere of BH acts like crank
  - Torque transmitted though poloidal field powers jet

- Blandford-Znajek process (Blandford & Znajek 1977)
  - Directly extract the BH rotating energy and convert to outward Poynting flux
  - Consider force-free limit (MHD Penrose process is similar mechanism)

Jet formation/acceleration mechanism

- In ideal MHD limit (infinite conductivity), plasma flow (motion) is connected with magnetic field
- The rotation of accretion disks or compact objects (BHs / NSs) twisted up the magnetic field into toroidal components



Pulsar magnetosphere

Collapsing, magnetized supernova core (GRBs)

Magnetized accretion disks around neutron stars and black holes

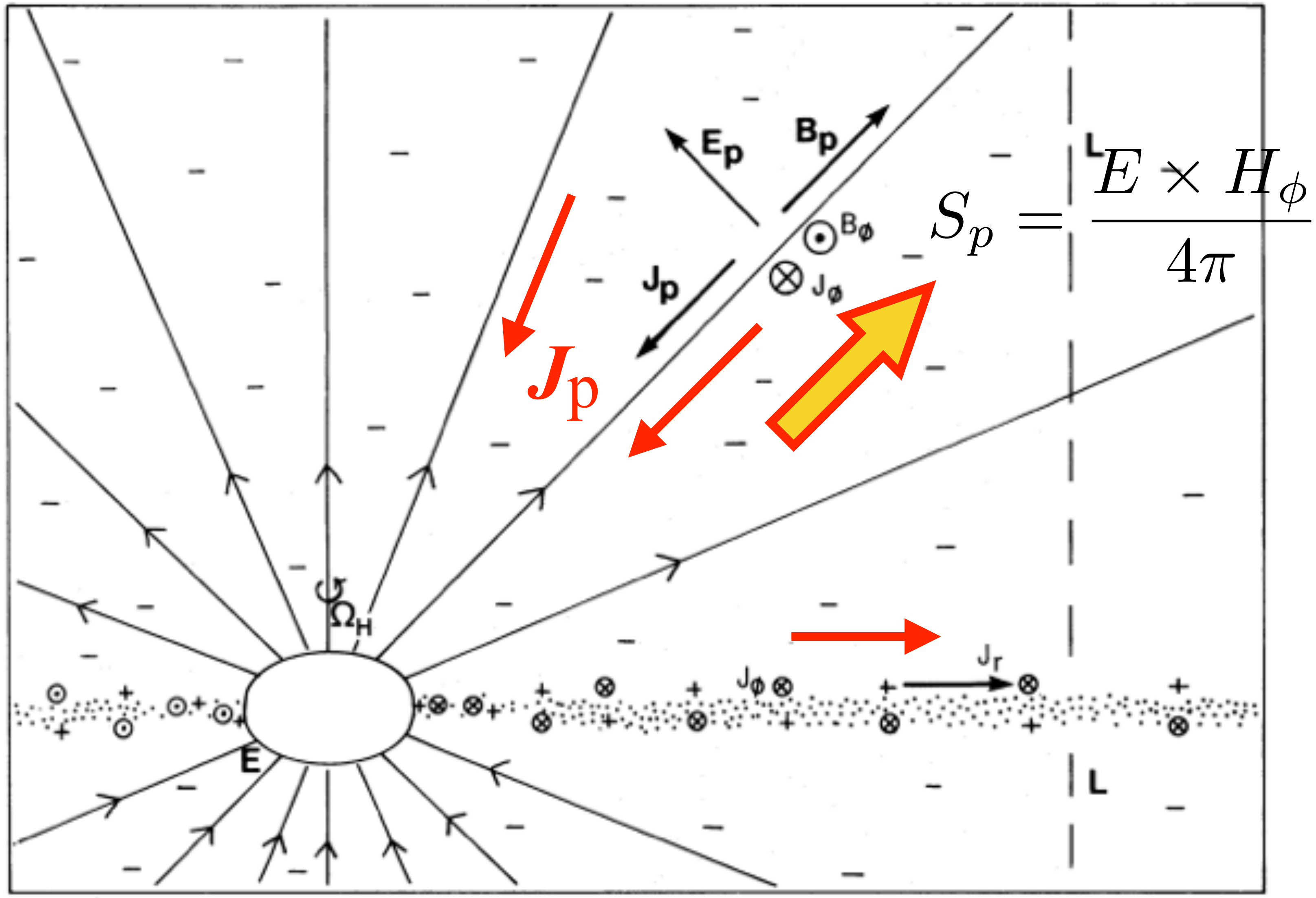
Magnetospheres of rotating black holes

Courtesy to David Meier

Blandford-Znajek Process

Blandford & Znajek (1977)

condition at infinity $H_\phi = -2\pi\Omega_F B^r \sqrt{\gamma}\sin\theta$



$$S_p = \frac{E \times H_\phi}{4\pi}$$

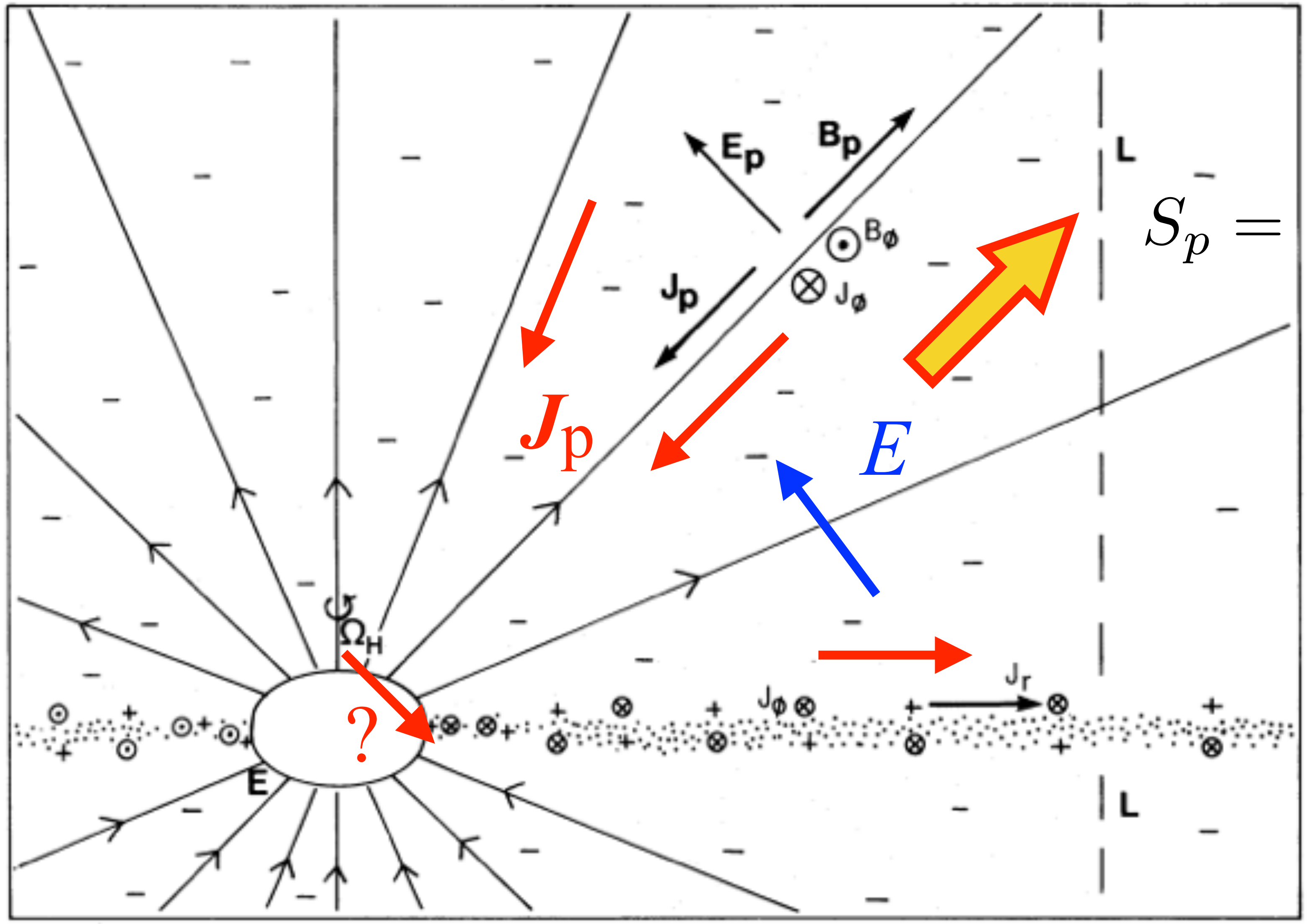
$$H_\phi = 2\pi(\Omega_F - \Omega_H)B^r\sqrt{\gamma}\sin\theta$$

at event horizon

- Kerr space-time
- Steady, axisymmetric
- Slowly rotating BH
- Split-monopole B field
- Force-free approximation (Electromagnetically dom.)

- Driving closed current system (load at infinity) => subject of strong criticism

Blandford-Znajek Process

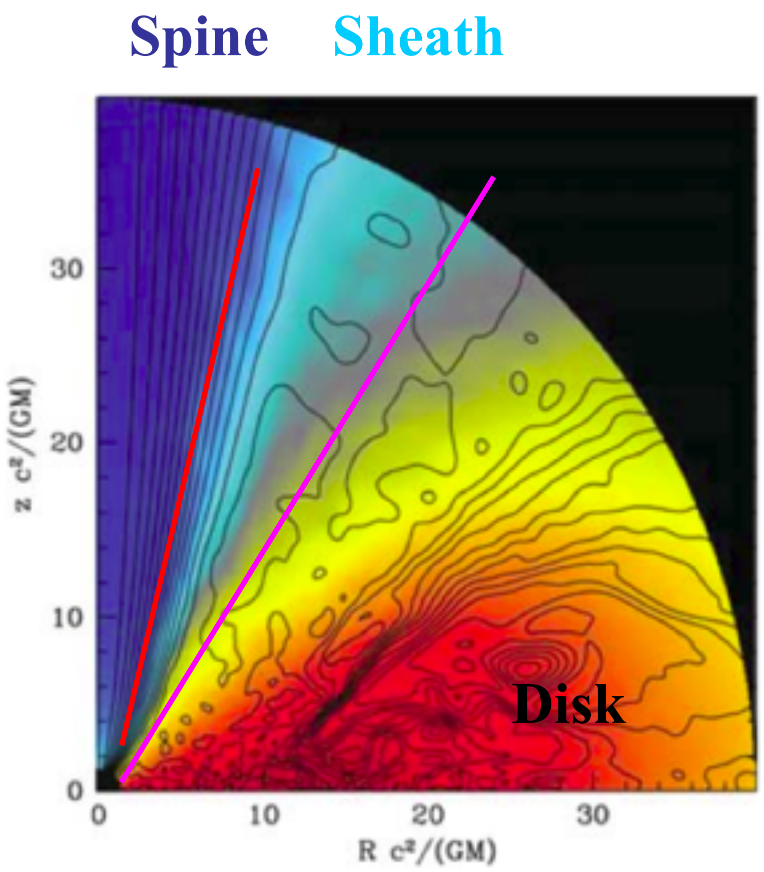


$$S_p = \frac{E \times H_\phi}{4\pi}$$

- Horizon is assumed as a rotating conductor (such as Membrane Paradigm). Ohmic dissipation increases BH entropy (Thorne et al. 1986; Penna et al. 2013)
- But the horizon is causally disconnected (Punsly & Coroniti 1989)
- Current driving mechanism is unclear (pair creation gap?)

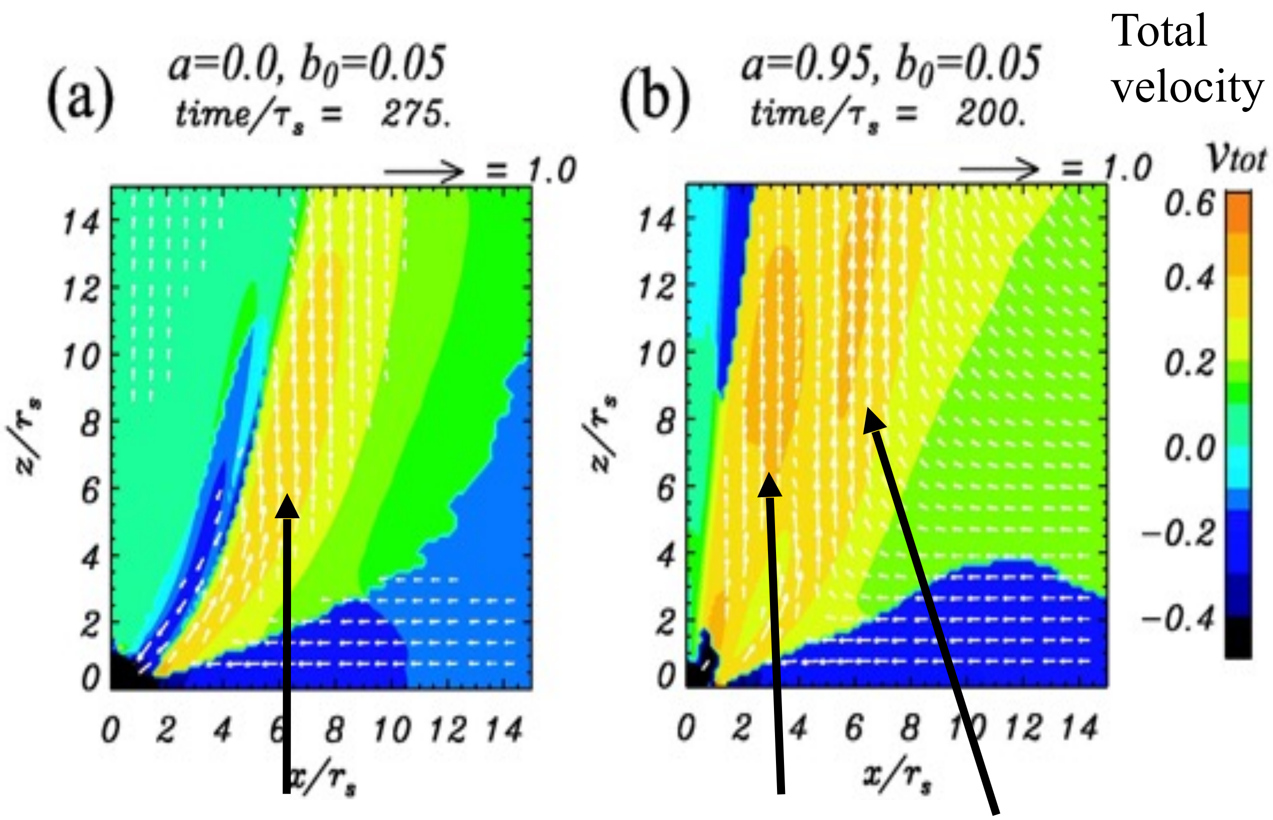
Relativistic Jets Formation from GRMHD Sim.

- Many GRMHD simulations of jet formation (e.g., Hawley & Krolik 2006, McKinney 2006, Hardee et al. 2007) suggest that
  - a jet spine (Poynting-flux jet) driven by the magnetic fields threading the ergosphere via MHD process or Blandford-Znajek process
  - may be surrounded by a broad sheath wind driven by the magnetic fields anchored in the accretion disk (mildly-relativistic wind).
  - High magnetized flow accelerates Γ >>1, but most of energy remains in B field.



Density distribution
(McKinney 2006)

Non-rotating BH **Fast-rotating BH**



Disk Jet/Wind **BH Jet** **Disk Jet/Wind**

(Hardee, Mizuno & Nishikawa 2007)

Jet Energetics

Gravity, Rotational energy (BH or accretion disk)

Efficient conversion to EM energy

Poynting flux (magnetic energy)

Easy to get ~ equipartition,
hard to get full conversion

Jet kinetic energy

Magnetic field is a medium for a transmission not a source

Jet Collimation

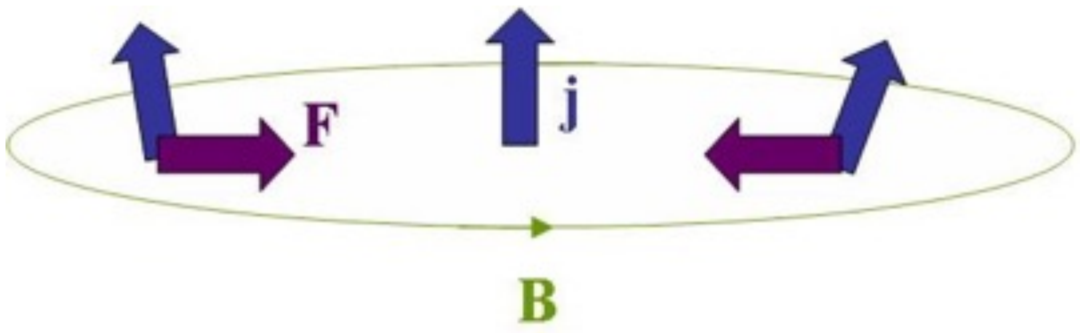
- Jet is produced by MHD process near the central objects and magnetic field is tightly tied (toroidal field is dominated)
- Lorentz force >> plasma pressure & inertia

⇒Huge tension force of wound up magnetic field (hoop stress) compress the flow towards the axis (self-collimation)?

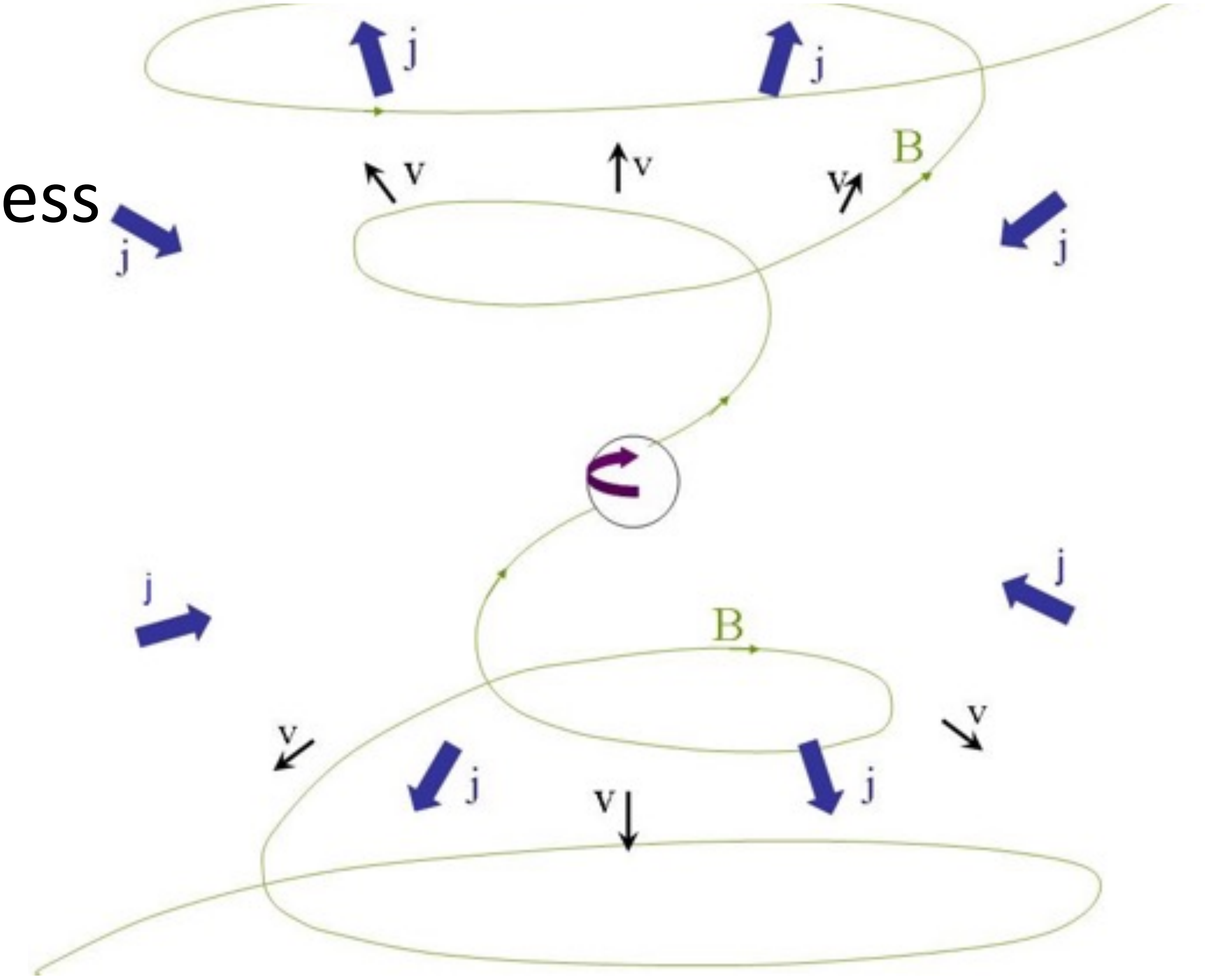
⇒Answer: No!

- In the current closure region, the force acts to de-collimation
- Need external confinement

Magnetic hoop stress



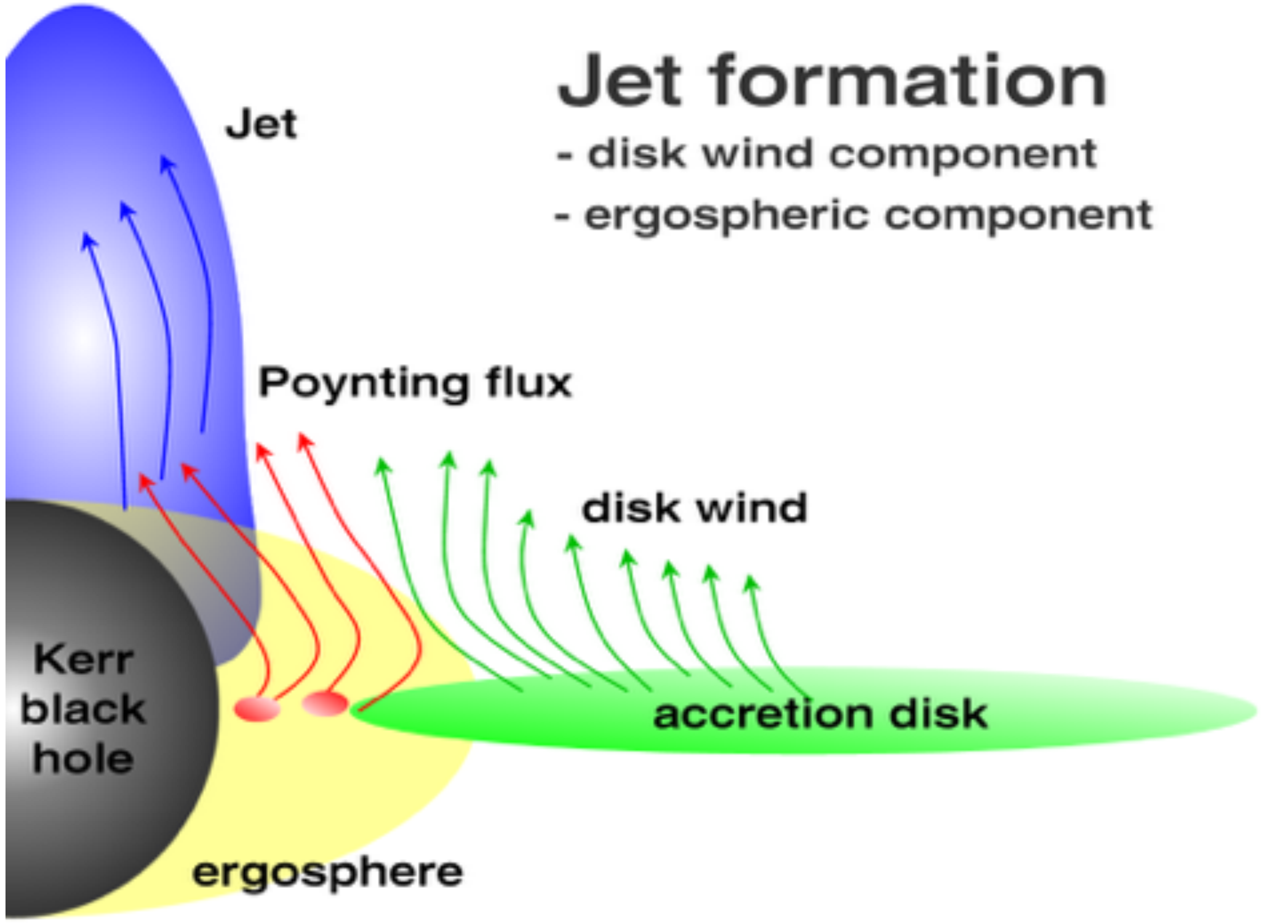
$$\mathbf{F} = \frac{1}{c}\mathbf{j}\times\mathbf{B}$$

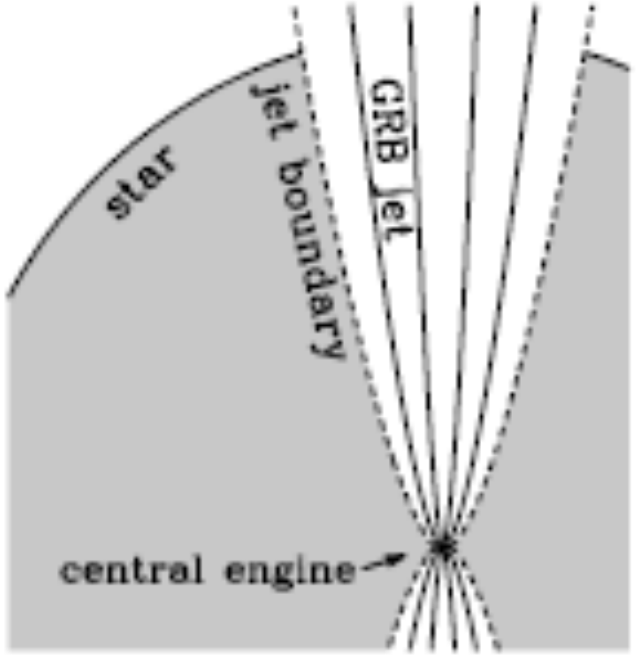


External Confinement

- In BH - accretion disk systems, the relativistic outflows from the black hole and the internal part of the accretion disk could be confined by the mildly-relativistic magnetized wind from the outer parts of the disk.

- In GRBs, a relativistic jet from the collapsing core pushes its way through the stellar envelope (confinement).





Collimation vs Acceleration

- For jet collimation, external confinement is necessary
- Without external confinement, the flow is near radial and acceleration stops at an early stage (Tomimatsu 1994; Beskin et al. 1998)
- The gas pressure profile of external confinement medium is the important parameter
- The spatial distribution of confining gas pressure determines the shape of the jet flow boundary, magnetic field configuration and acceleration rate (Tchekovskoy et al. 2009, 2010; Komissarov et al. 2009; Lyubarsky 2009,2010).

- Optimal collimation ⇔ pressure decrease slowly along jets
- Optimal acceleration ⇔ pressure decrease rapidly along jets
  => Collimation and acceleration of jet are related (poloidal) magnetic field configuration

Effects of external confinement

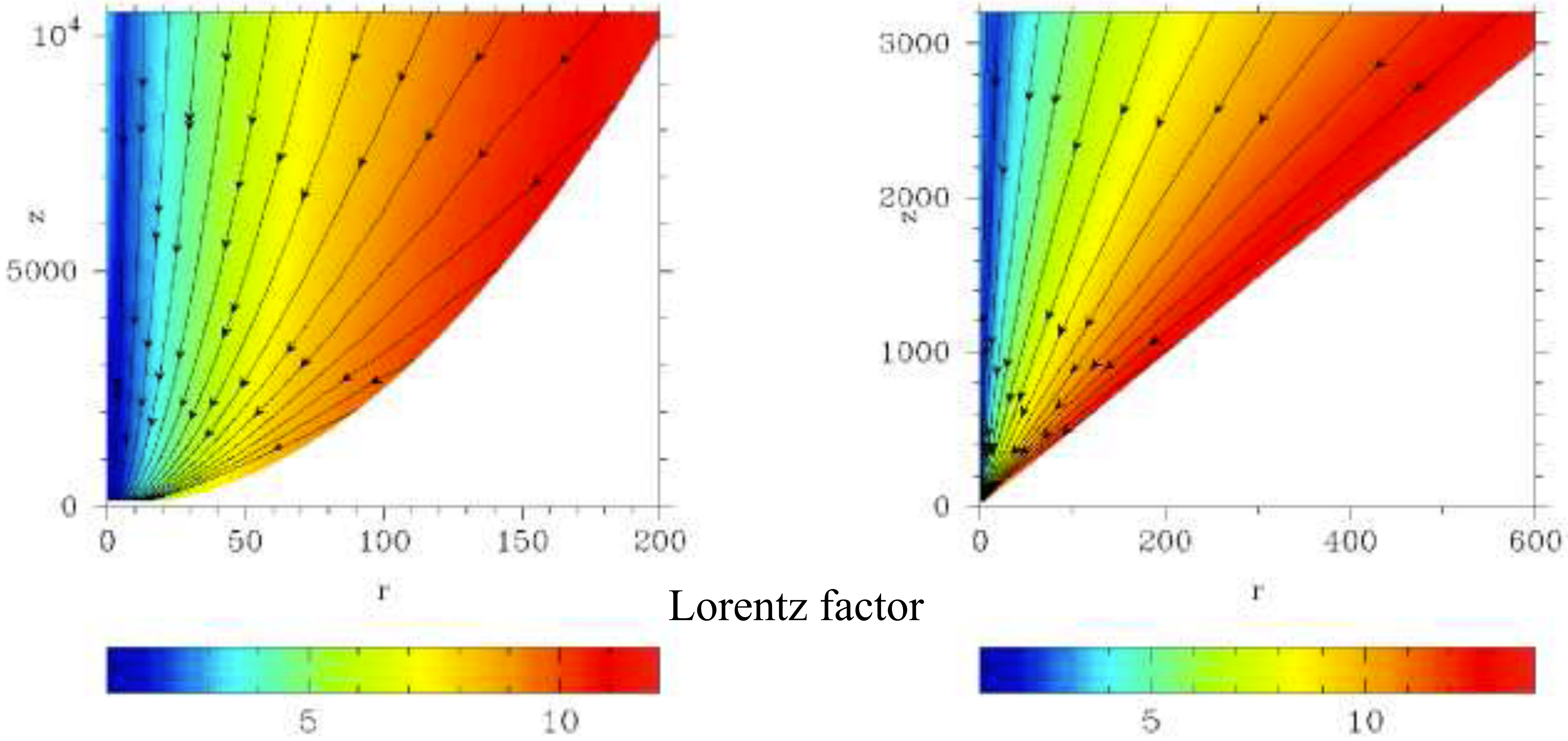
2D RMHD simulations (Komissarov et al. 2009)

Parabolic ($z \propto r^2$)

Acceleration: slow, collimation: OK

Conical ($z \propto r$):

Acceleration: fast, collimation: X



Lorentz factor

- Some part of jets can convert Poynting flux to Kinetic Energy but most can’t.
- Energy conversion is too slow to become kinetic energy dominated, it is unreasonably long distance = inconsistent of observations.
- We need to consider some sort of dissipation (rapid energy conversion)

Global structure of M87 jet

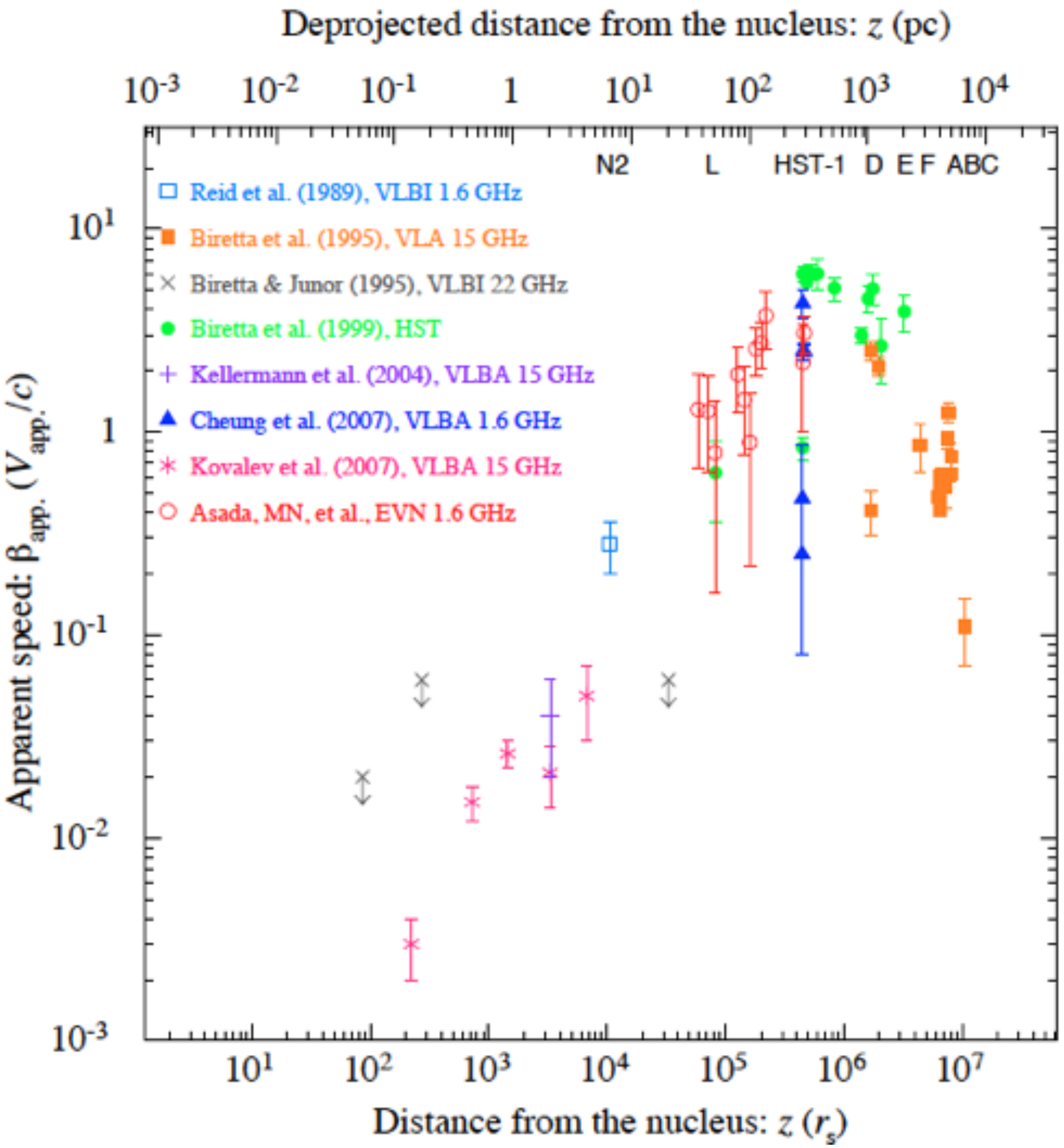
- In M87 jet, the asymptotic acceleration from non-relativistic (0.01c) to relativistic speed (0.99c) occurs over 10^{2-5} r_s

- This is very slow acceleration = consistent with theoretical results?

- The absence of bulk-Comptonization spectral signatures in blazars implies that Lorentz factors >10 must be attained at least ~1000 r_g (Sikora et al. 05).

- But according to spectral fitting, jets are already matter-dominated at ~1000 r_g (Ghisellini et al 10).

Transition of Sub- to super-luminal motion in M87 jet



Asada & Nakamura (2014)

Dissipation in the Jet

- Time-dependent energy injection to jet

=> Internal shocks in jets

- Sudden change of confined external medium spatial profile

=> Recollimation shock/ rarefaction acceleration

- Magnetic field reversal or deformation of ordered magnetic field

=> Magnetic reconnection

- MHD Instabilities in jets
  - Kelvin-Helmholtz instability at jet boundary
  - Current-Driven Kink instability at jet interior

  => Turbulence in the jets and/or magnetic reconnection?

Dissipation in the Jet: Energetics

- Tapping kinetic energy
  - Internal shock
  - Recollimation shock
  - Kelvin-Helmholtz instability

- Tapping magnetic energy
  - Rarefaction acceleration
  - CD kink instability
  - Magnetic reconnection

Prefer dissipation mechanism for Poynting-dominated jet (conversion from Poynting flux to Kinetic energy)

Here I skip detail RMHD simulation work about magnetic dissipation

Predicting the realistic BH shadow image

- Milimetre (submm)-VLBI of EHT will be achieved the event horizon scale observation (BH shadow image) in near future
- Ingredients for realistic theoretical image of BH shadow

1. Plasma behaviour surrounding BH

   Consider time evolution of accreting matter onto BH and formation of relativistic jets

2. Radiation process

   Consider GR effects (geodesic, redshift), thermal/non-thermal radiation process, optical thickness etc.

3. BH spacetime
4. VLBI array configuration and schedule

- Tools: General Relativistic MHD code + General Relativistic Radiation Transfer code + synthetic imaging

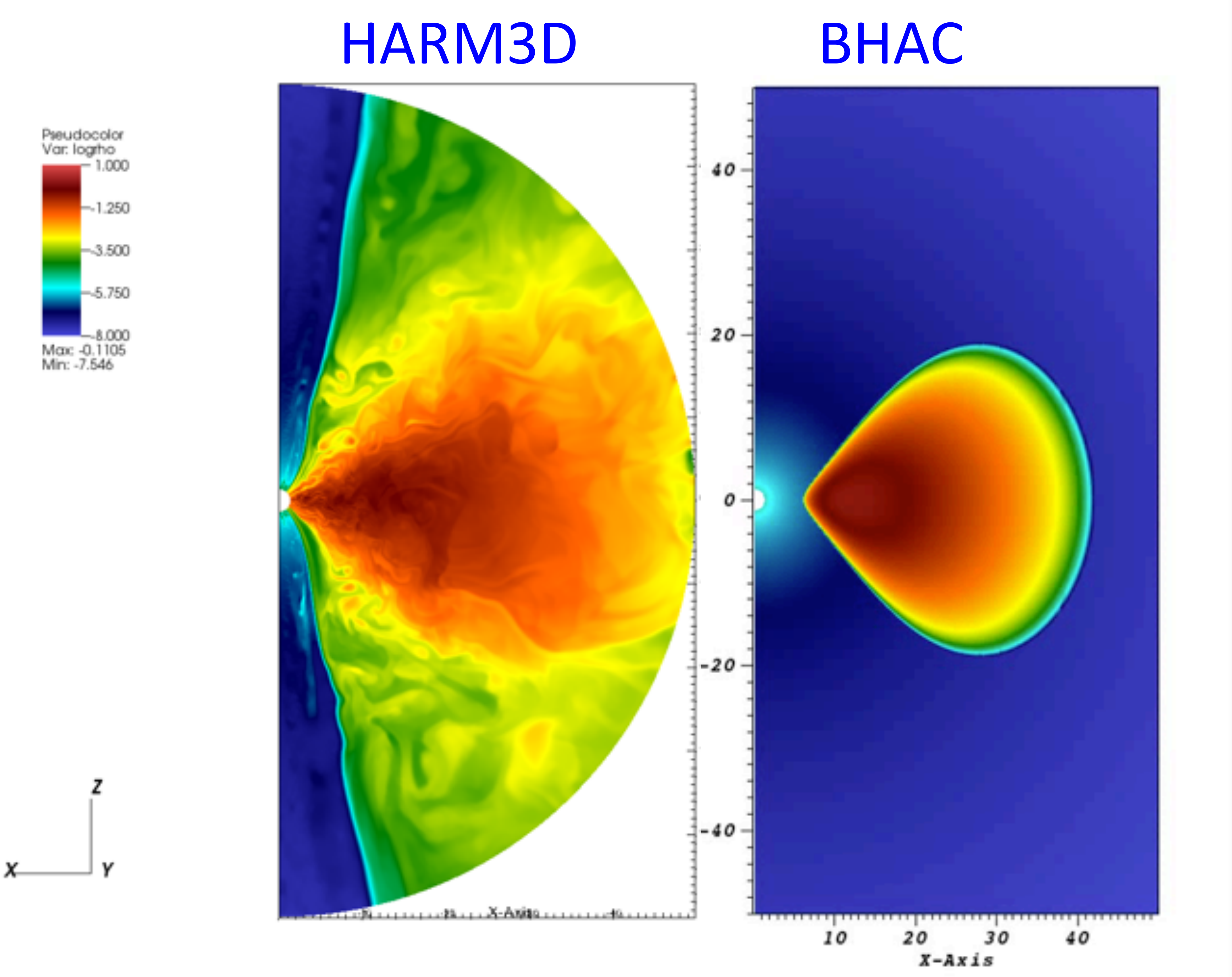
Fishbone & Moncrief (1976) torus model

Validation (global structure)

Porth et al. (2017)

- $a = 0.9375$
- $r_{in} = 6$
- $r_{max} = 12$
- $A_\phi \propto (\rho/\rho_{max} - 0.2)$
- $\beta = (p_{g,max}/p_{mag,max}) = 100$
- Coordinates: Logarithmic KS
- $r \in [0.96 r_H, 50M], \theta \in [0, \pi]$
- $\Gamma = 4/3$
- $\rho_{atm} = 10^{-5}\, r^{-1.5}$
- $p_{atm} = 3.3 \times 10^{-8}\, r^{-2.5}$

- HARM3D (*Noble et al. 2009*) simulations (in 2D setting) from Moscibrodzka
- Very good quantitative and qualitative agreement



Logarithmic densities at t=2000 M in resolution 512 x 512.
PPM reconstruction, LF Riemann solver, Flux-CT

*actually for BHAC: $r \in [0.96 r_H, 2500M$

Fishbone & Moncrief (1976) torus model

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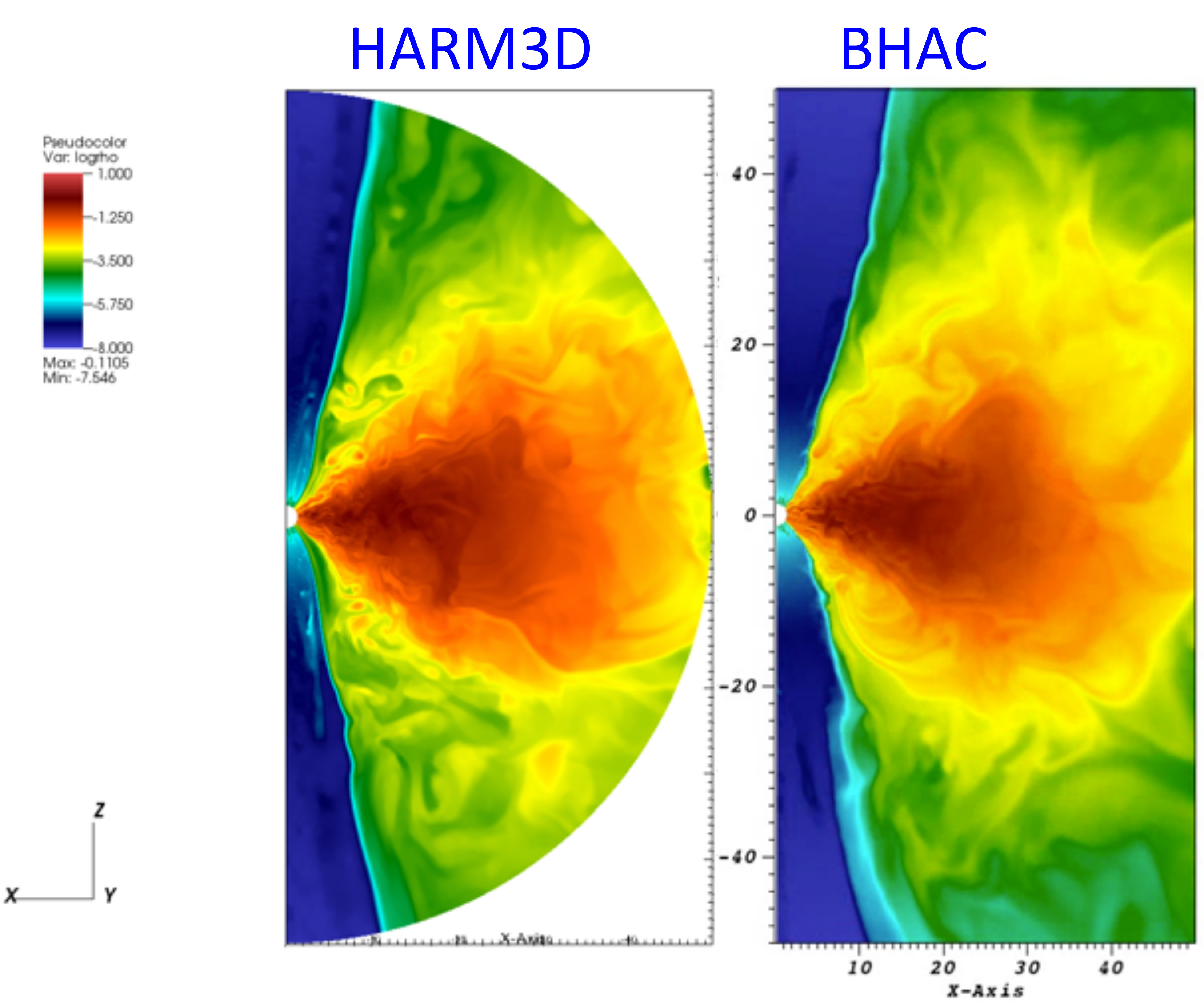

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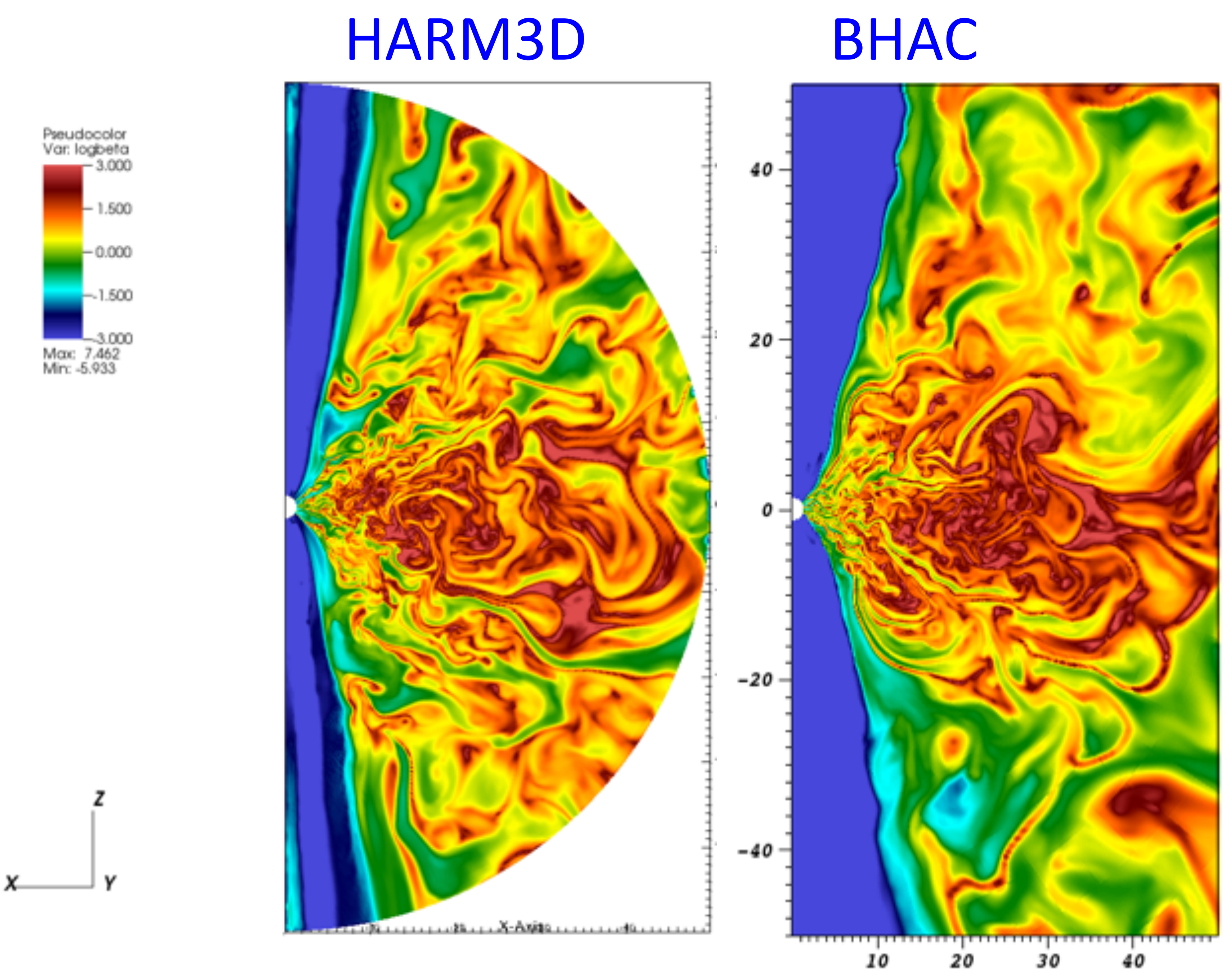


Logarithmic plasma beta at t=2000 M in resolution 512 x 512. PPM reconstruction, LF Riemann solver, Flux-CT

*actually for BHAC: $r \in [0.96 r_H, 2500M$

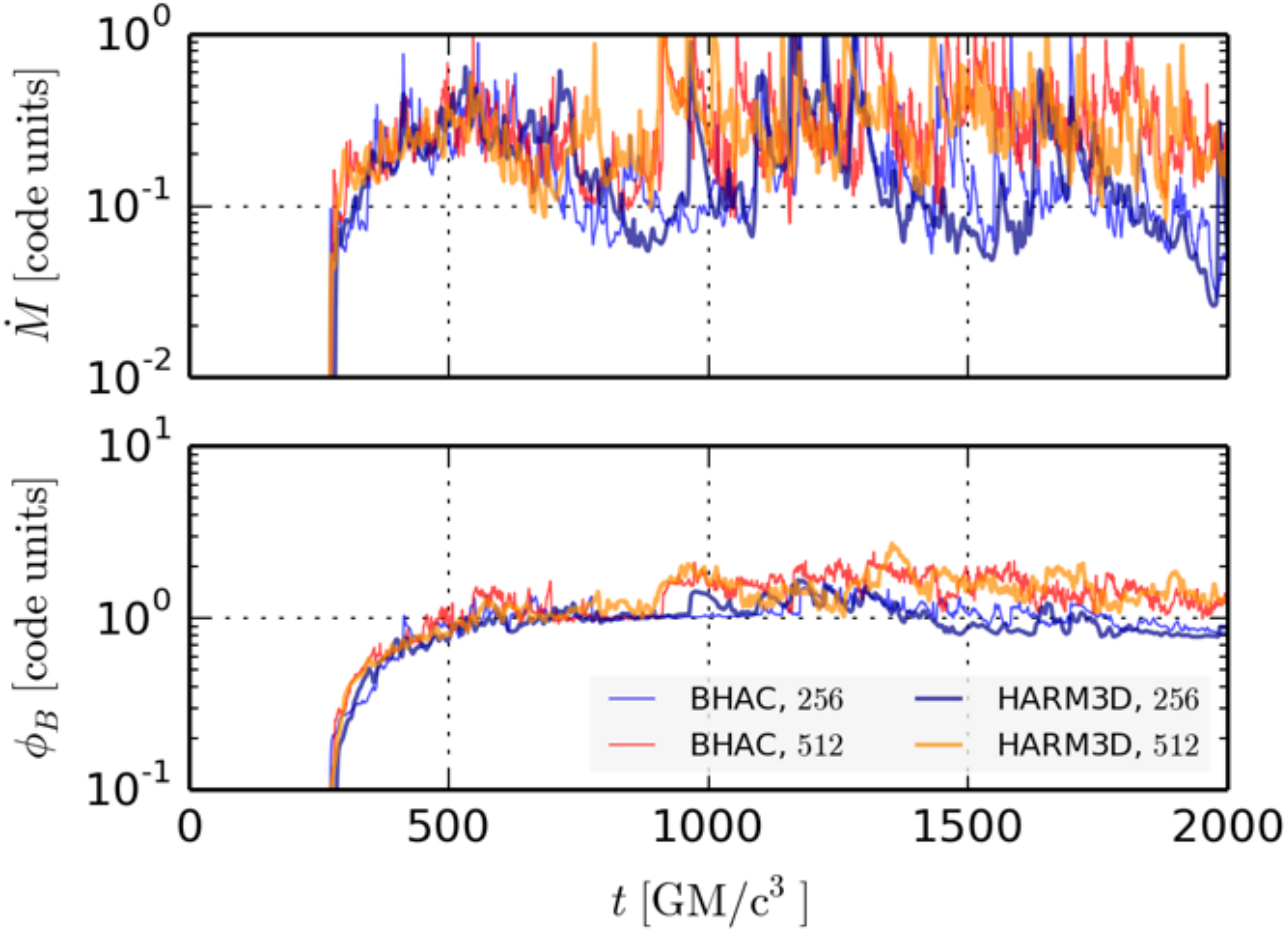
Validation (accretion rate)

Porth et al. (2017)

Accretion rates and magnetic flux threading the horizon in BHAC & HARM3D

Mass accretion rate

Magnetic flux accretion rate

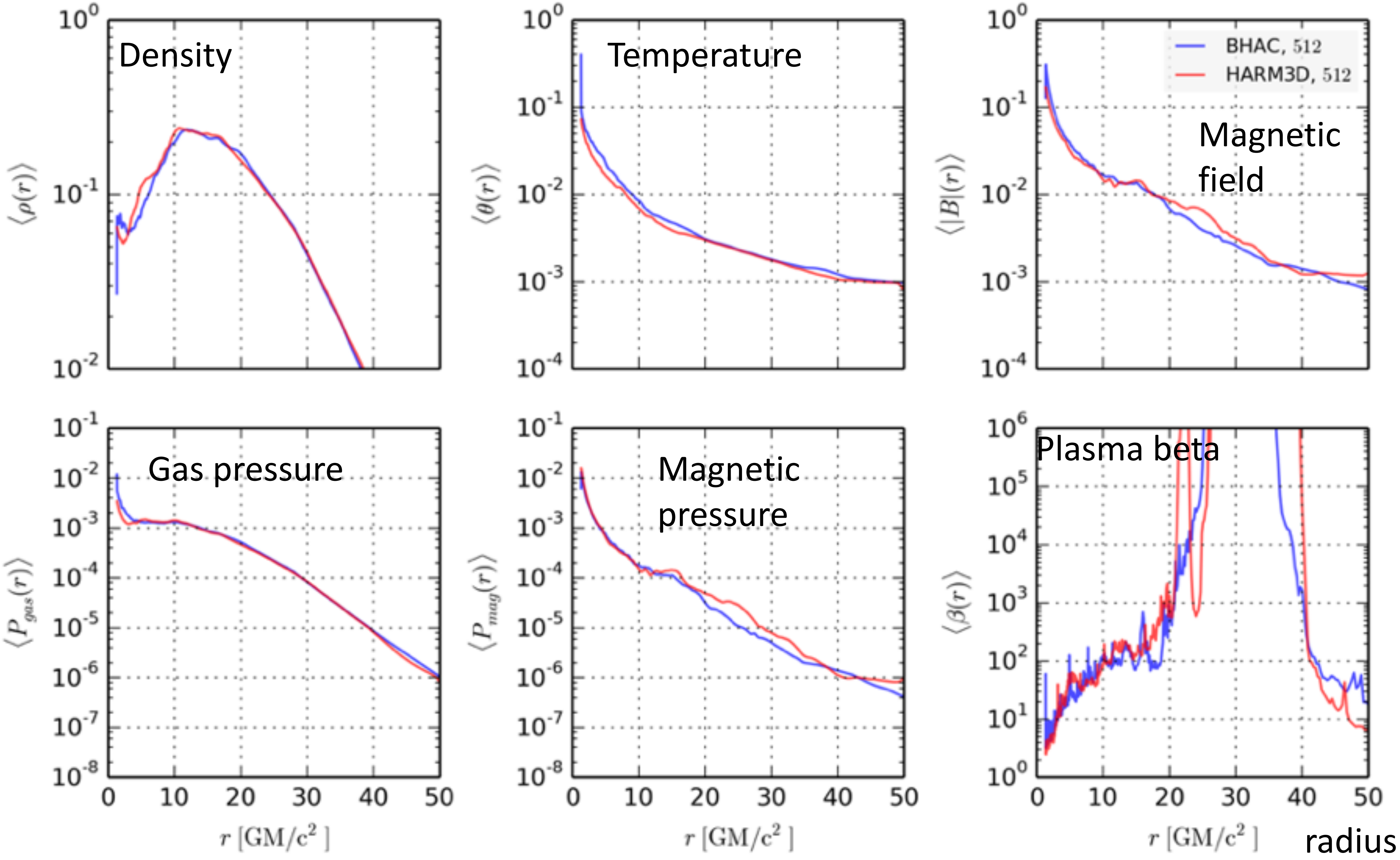


- Double resolution => roughly double acc. rate and flux on BH (2D!)
- Very good quantitative and qualitative agreement

Validation (accretion rate)

Porth et al. (2017)

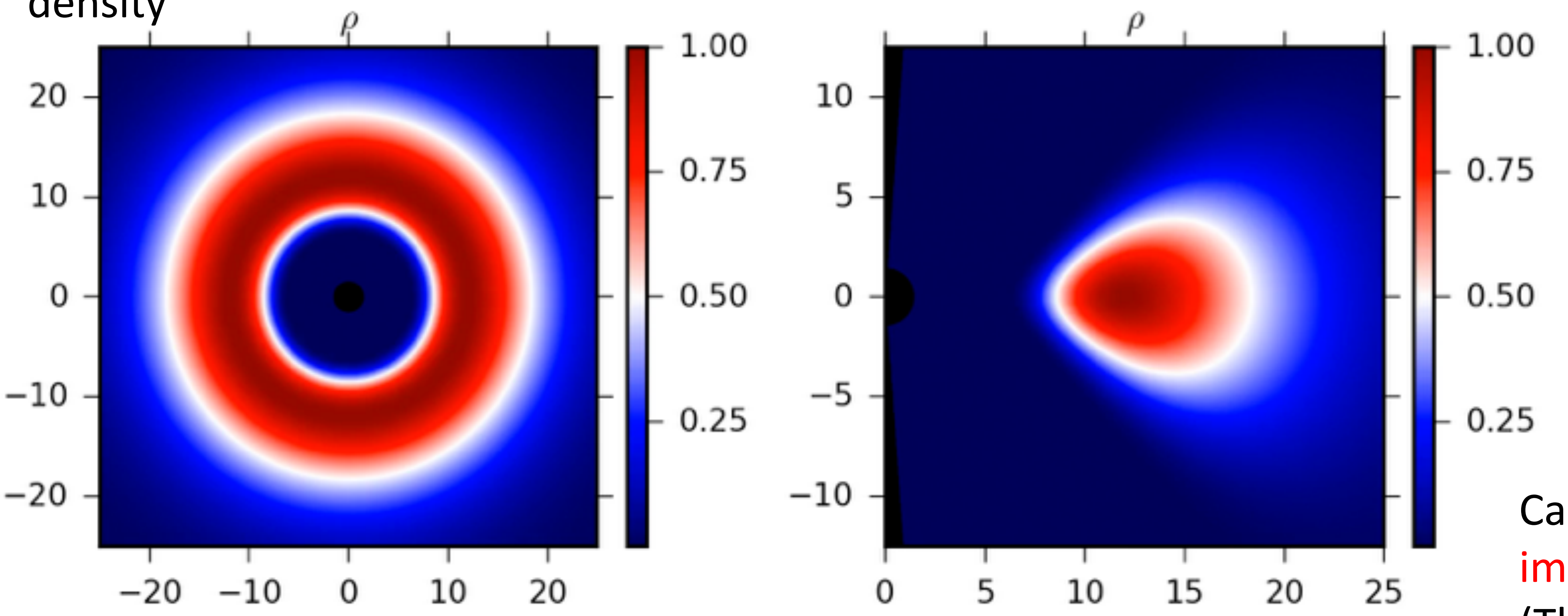
Azimuthal averaged disk profiles of quantities of interest in BHAC & HARM3D



- Very good quantitative and qualitative agreement

3D GRMHD simulations of magnetized torus

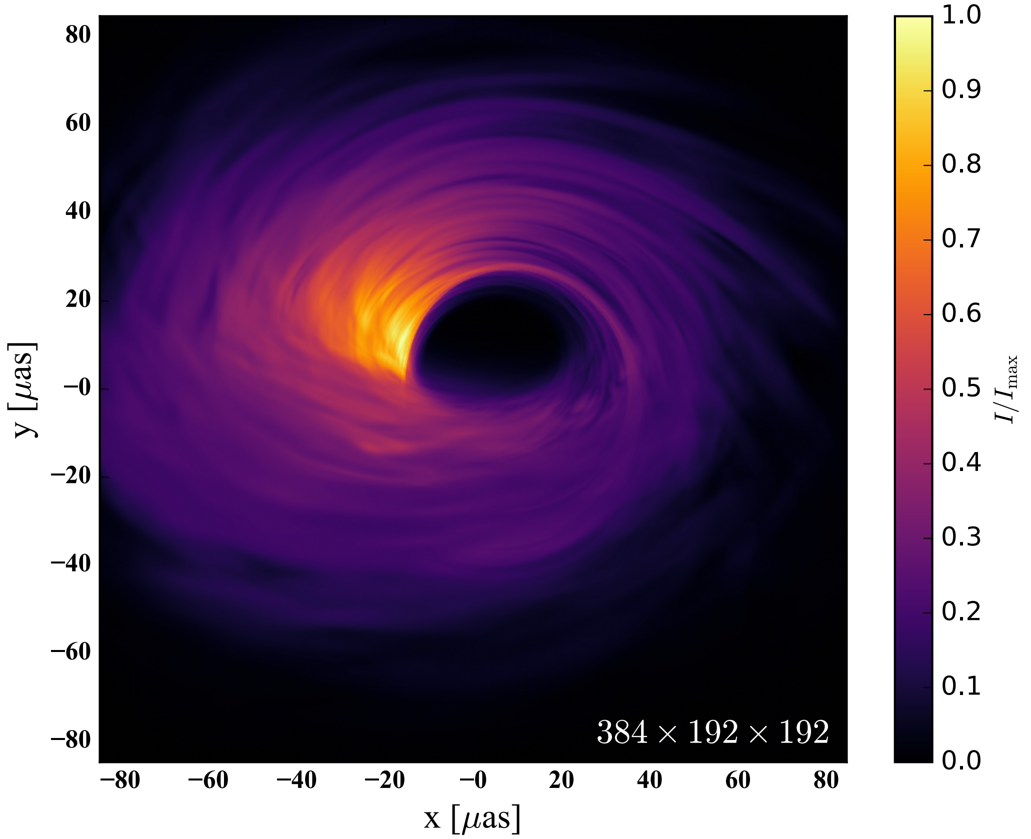
density



Porth et al. (2017)

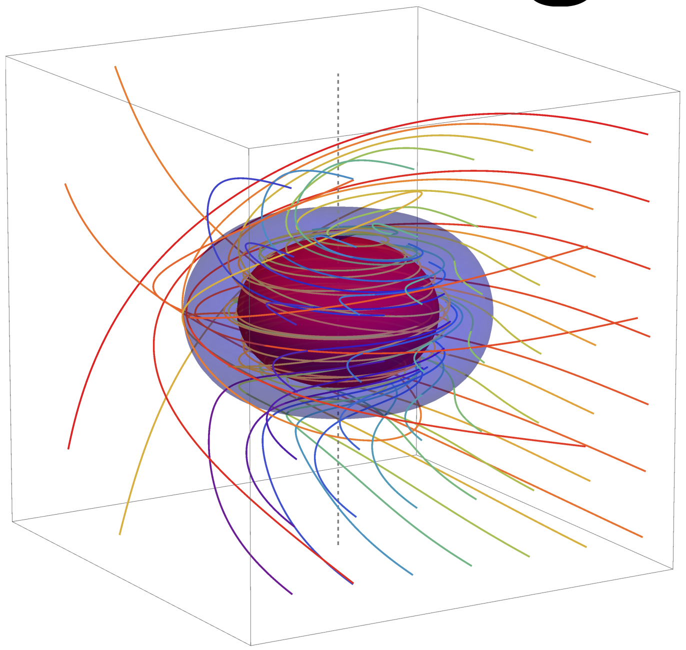
- Initial: Accretion torus + weak single magnetic field loop
- Inside torus becomes turbulent by MRI
- Poynting flux dominated jet is developed near the axis

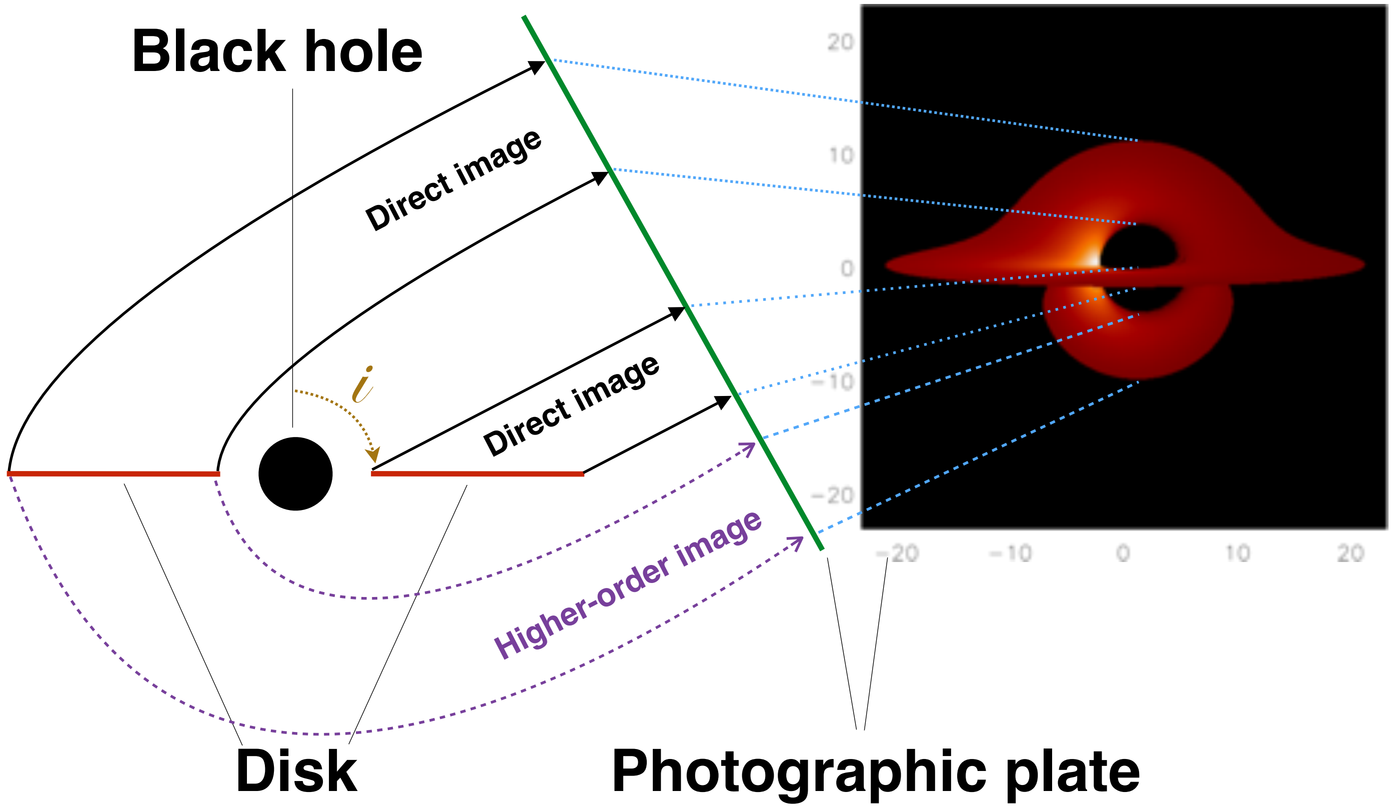
Calculated Radiation image by GRRT code (Thermal synchrotron total intensity)



- We can obtain BH shadow image, spectrum, light curve (+ polarization) via 3D GRMHD simulations

Strong GR: The Black Hole Shadow





How to Image a Black Hole

Bardeen (1973)
Luminet (1979)
Falcke et al. (2000)
Takahashi (2004)
etc.

Shadow diameter: Non-spinning (a=0)

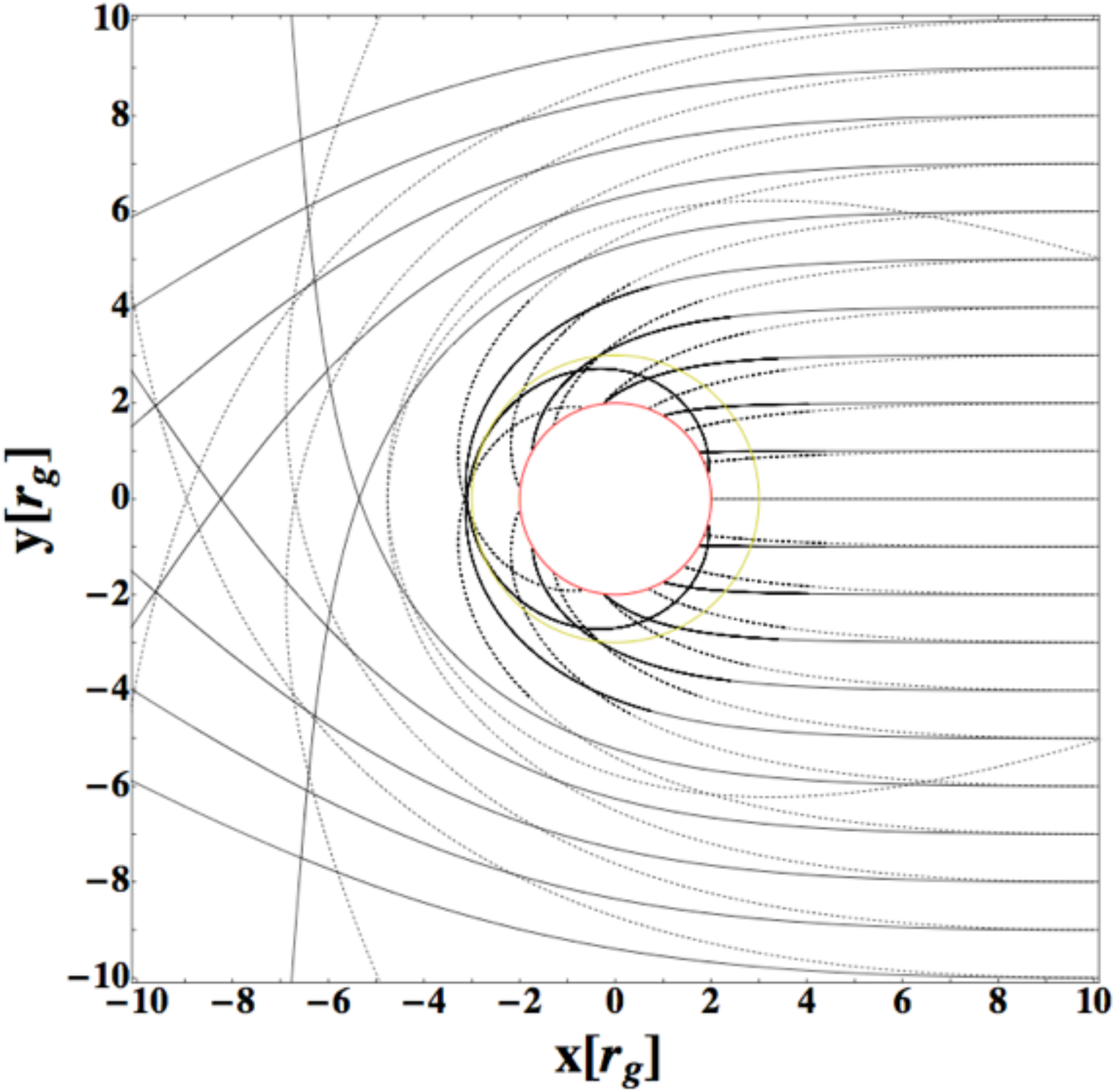
$$D_{sh} = \mathrm{sqrt}(27) * R_{sch}$$

Spinning (a=1)

$$D_{sh} = 9/2 * R_{sch}$$

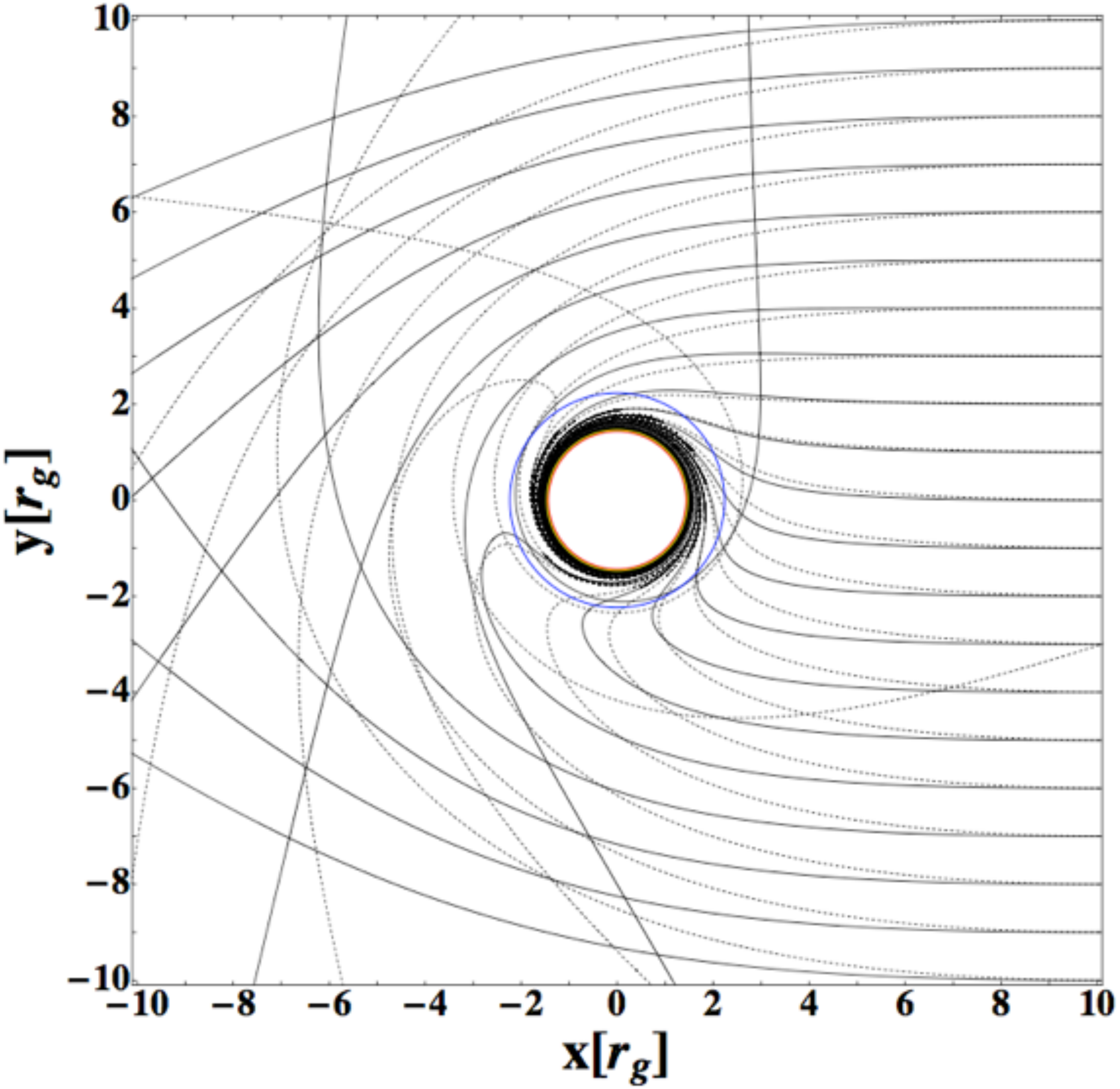
- Shadow size and shape encodes GR (e.g., Johannsen & Psaltis 2010)

Schwarzschild Geodesics



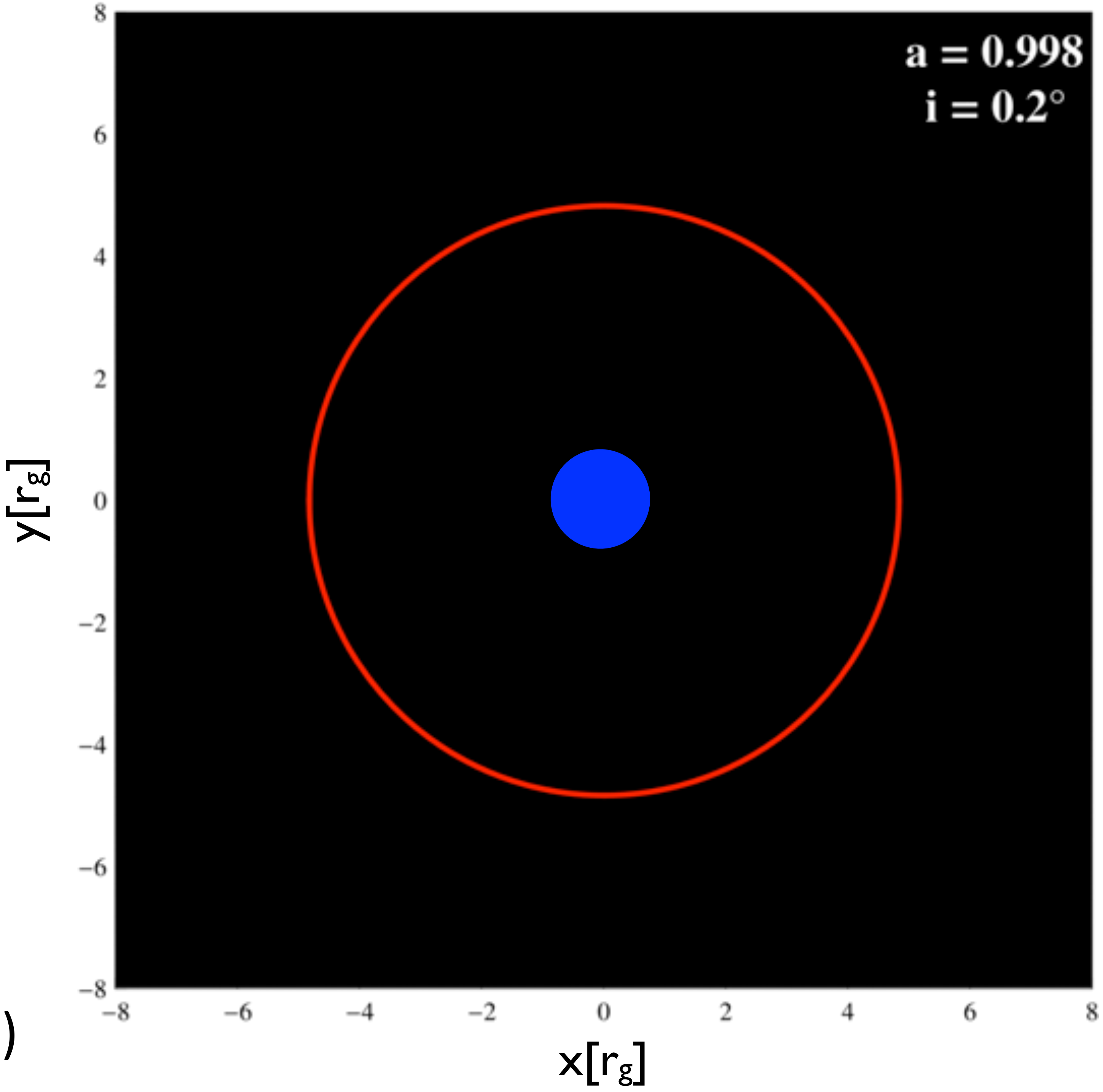
Kerr Geodesics

(a=0.998)



Black Hole Shadow

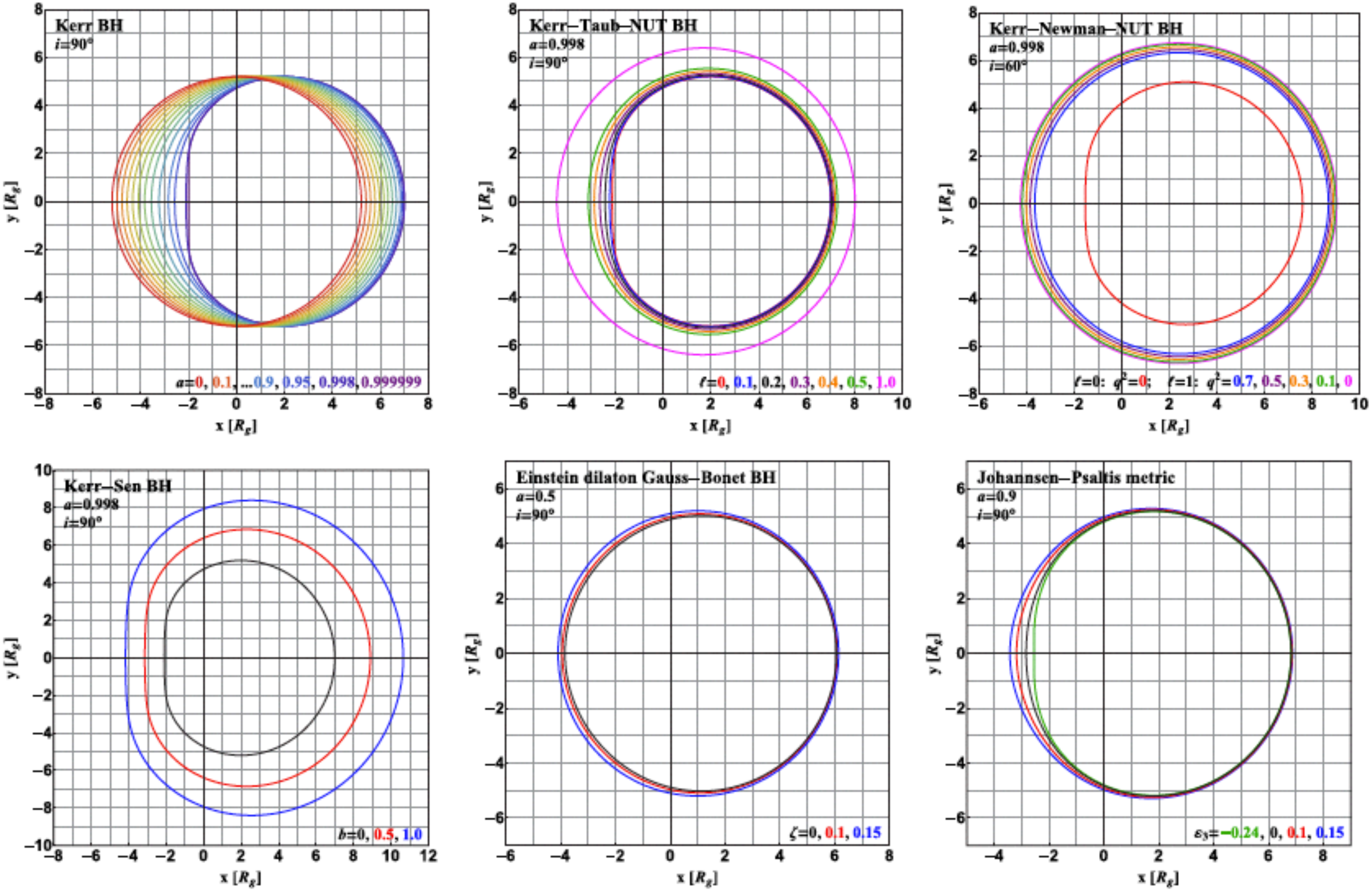
Black Hole shadow boundary curve in different inclination angle (Kerr BH with a=0.998)



Movie by Z. Younsi (BHOSS code)

Shadow industry: Different Spacetime

Variety of BH shadow boundary curve in different theory of gravity



From BHCam review paper by Goddi et al. (2017)

Which gravitational theory?

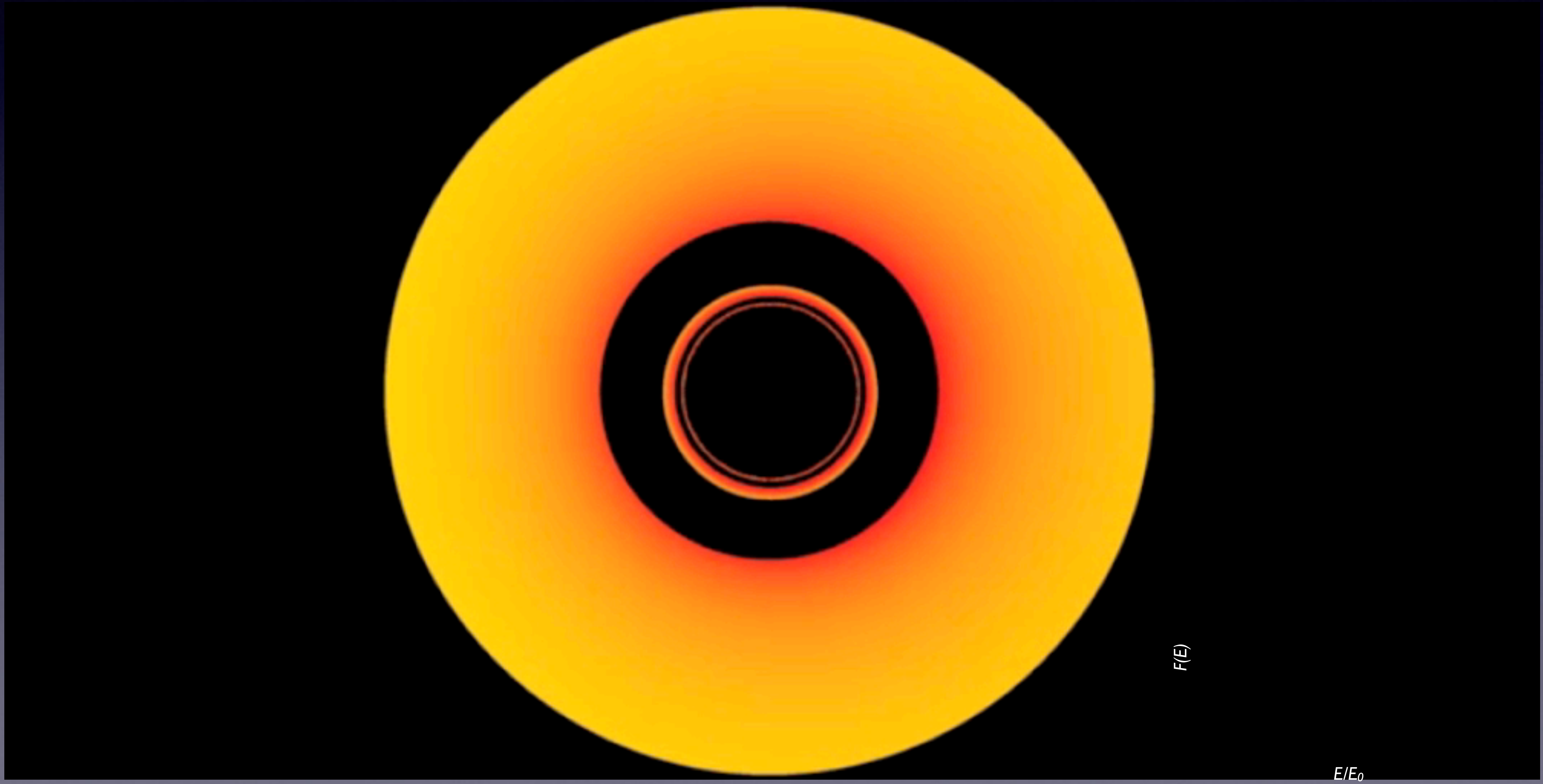
- Future mm/sub-mm VLBI observation of EHT will provide the first images of the BH shadow in our galactic centre, Sgr A* & M87.

- If the observations are sufficiently accurate, it will provide
  - the evidence for the existence of an event horizon
  - Testing the no-hair theorem in GR
  - Testing of GR itself against a number of alternative theories of gravity.

- Reasonable to use a model-independent framework which parametrises the most generic BH geometry though finite number of adjustable quantities.
- Recently new parametric framework of generic metric is proposed in spherically symmetric BH (Rezzolla & Zhidenko 2014) and in axisymmetric BH (Konoplya et al. 2016)

Optically Thick Accretion Torus

Movie by Z. Younsi (BHOSS code)

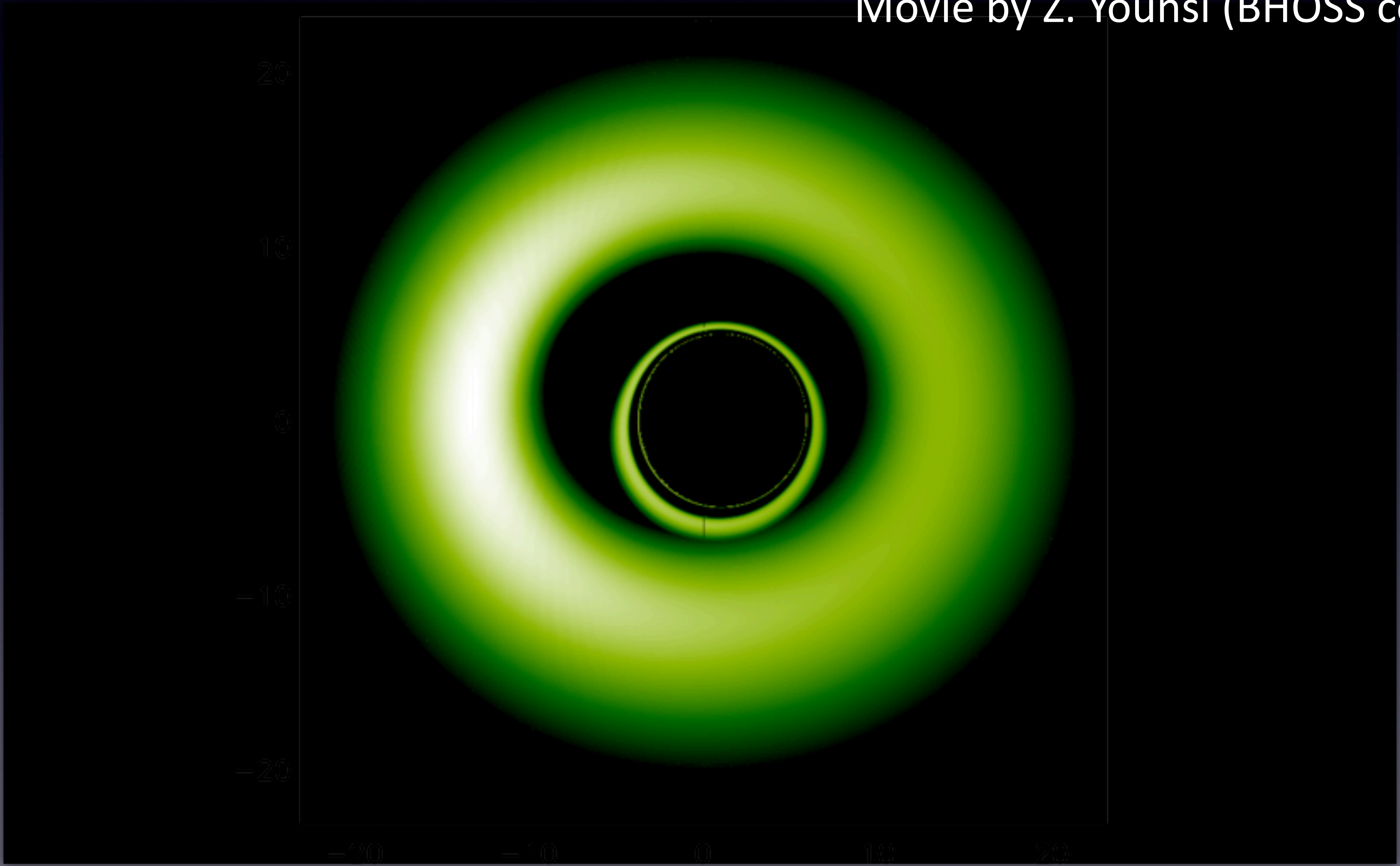


Energy shift

Emission line spectrum

Emission From Optically Thin Accretion Torus

Movie by Z. Younsi (BHOSS code)



Intensity

Dilaton Black Holes

- For first test, consider non-rotating Dilaton black hole.
  (coming from Einstein-Maxwell-dilaton-axion (EMDA) gravity which is the low energy limit of the bosonic sector of the heterotic string theory)

- When both the axion field and the BH spin vanish, such a BH is described by spherically symmetric metric

$$ds^2 = -\left(\frac{\rho - 2\mu}{\rho + 2b}\right) dt^2 + \left(\frac{\rho + 2b}{\rho - 2\mu}\right) d\rho^2 + (\rho^2 + 2b\rho) d\Omega^2$$

(Exact form)

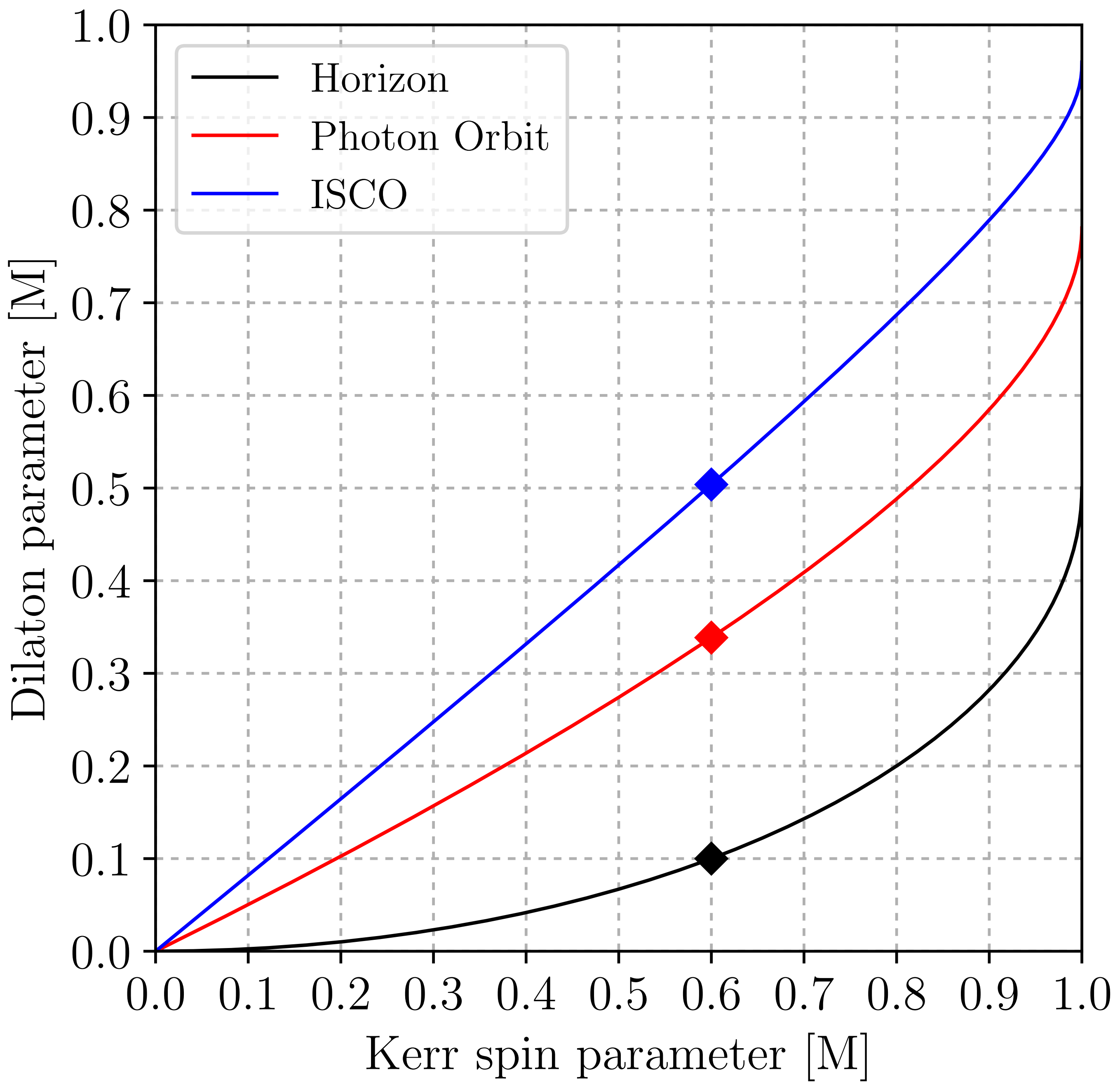
$r^2 = \rho^2 + 2b\rho, \quad M = \mu + b$ r: radial coordinate, M: ADM mass, b: dilaton parameter

- It is clear that if $b=0$, we reproduce Schwarzschild BH metric.

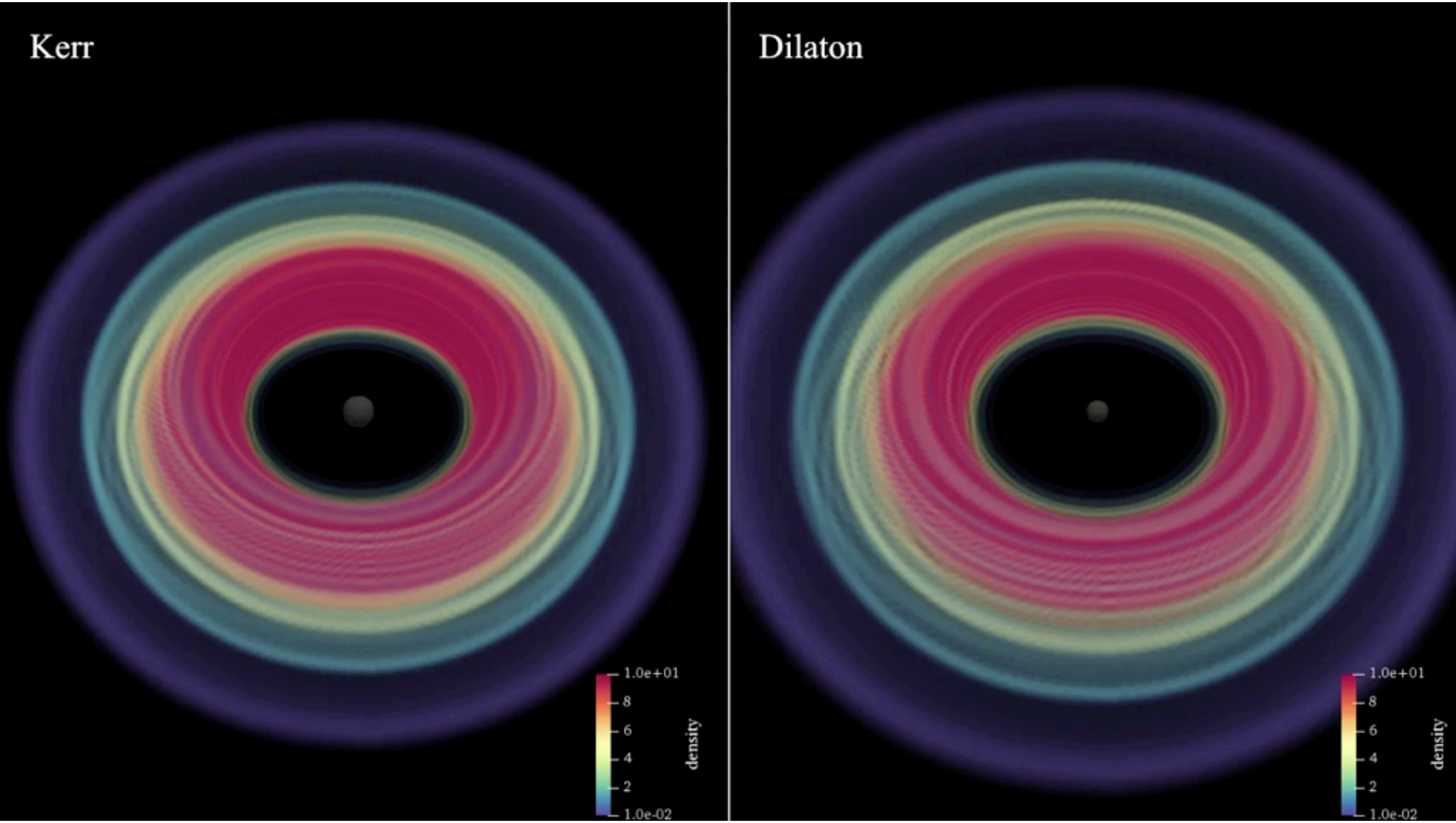
- Use Rezzolla & Zhidenko parameterized metric to describe non-rotating Dilaton BH metric

Dilation vs Kerr

- Does Dilation BH mimics Kerr BH?
- Three characteristic radius: horizon radius, photon orbit, ISCO
- Larger dilation parameter makes smaller horizon radius, Photon orbit, & ISCO
- Similar to Kerr spin parameter.
- How affects for plasma behaviour anc radiation signature (BH shadow image)?



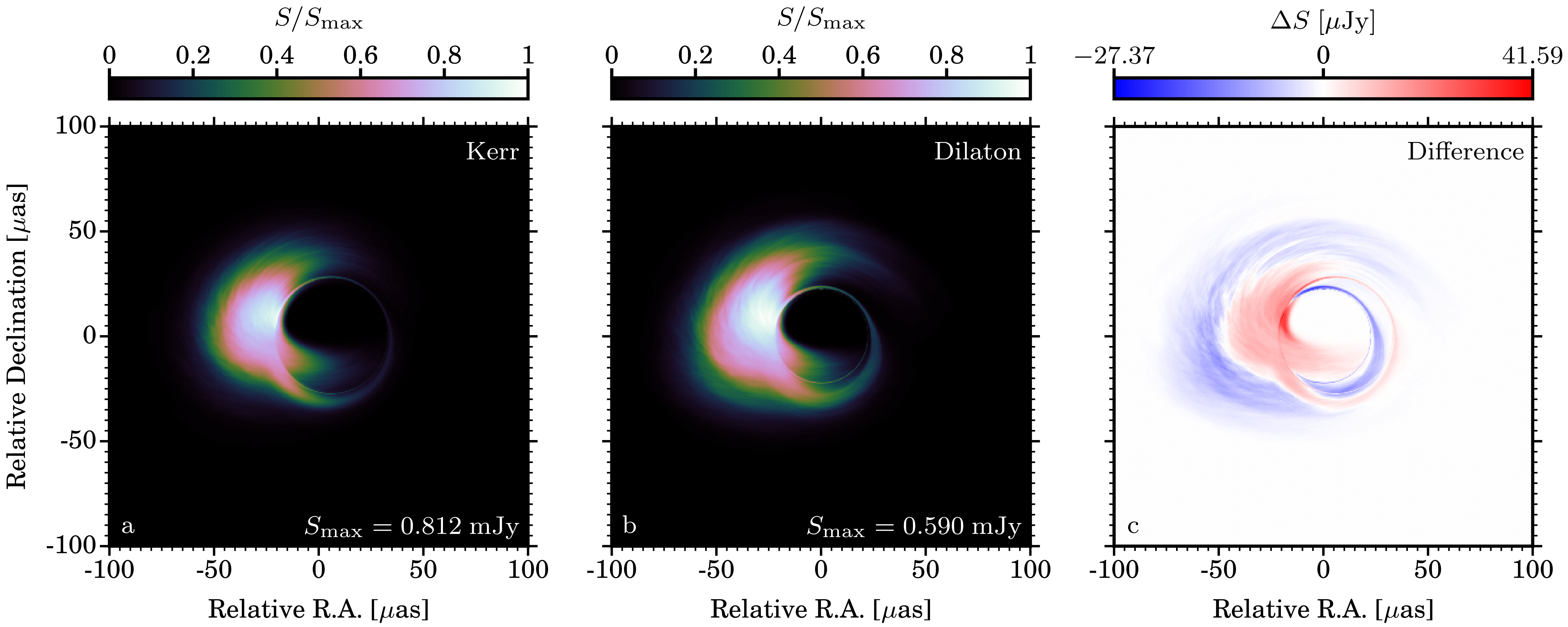
3D GRMHD simulations



- 3D GRMHD simulations of magnetized torus with a weak poloidal magnetic field loop accreting onto Kerr BH (a=0.6) & ISCO-matched dilaton BH (b=0.5)

BH shadow image

Intensity map @ 230GHz, i=60 deg, time-averaged (t=11000-12000M) by *BHOSS*

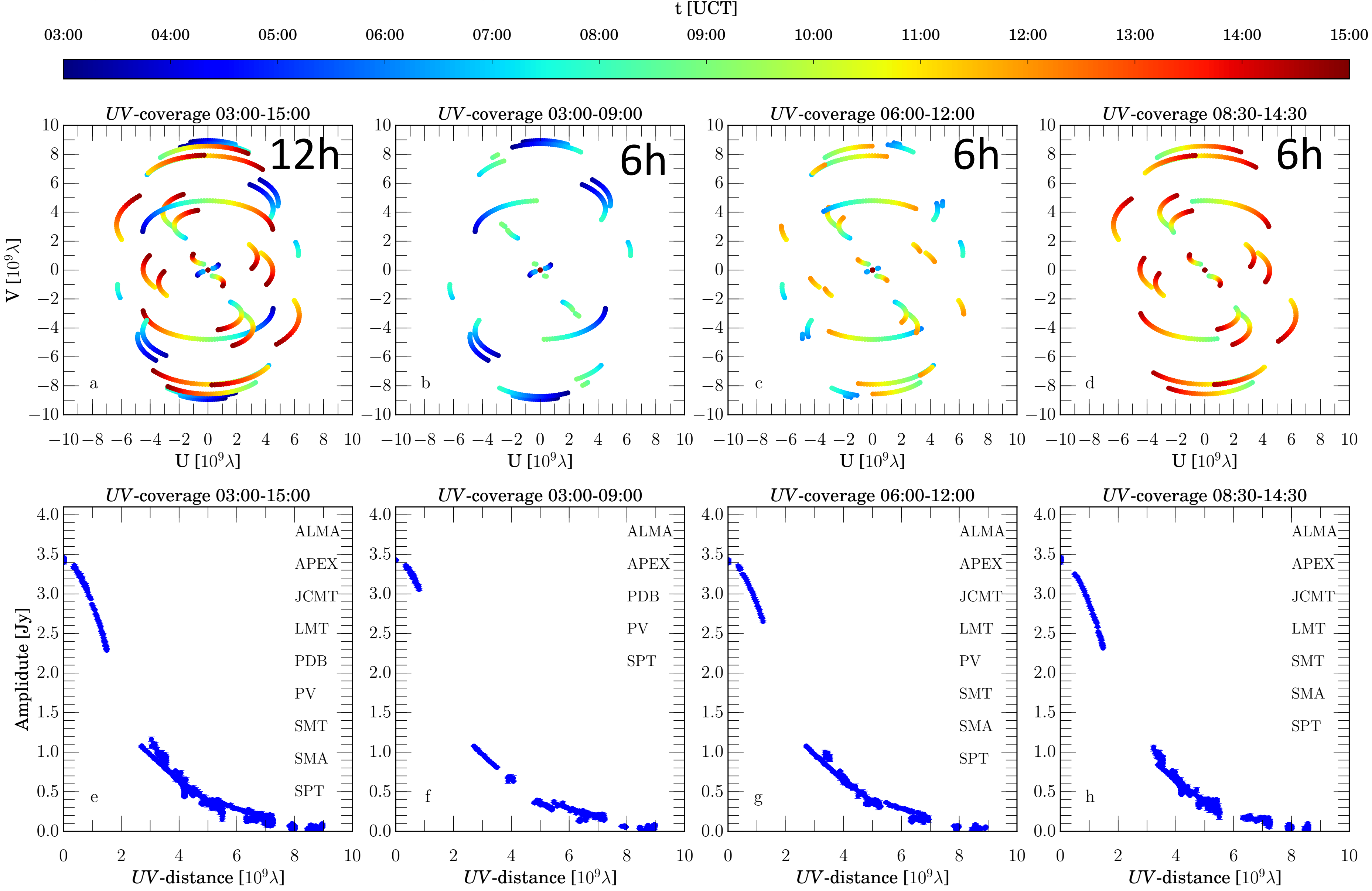


- Emission model (fixed $T_i/T_e = 3,\ \dot{M} \sim 10^{-9} M_\odot\,\mathrm{yr}^{-1}$)
- BH shadow image is quiet similar … but we see some difference
- Pixel-by-pixel difference shows smaller shadow size by dilaton BH (blue ring), and offset & asymmetry of shadow by Kerr BH (red ring)
- But this is “infinite-resolution images”

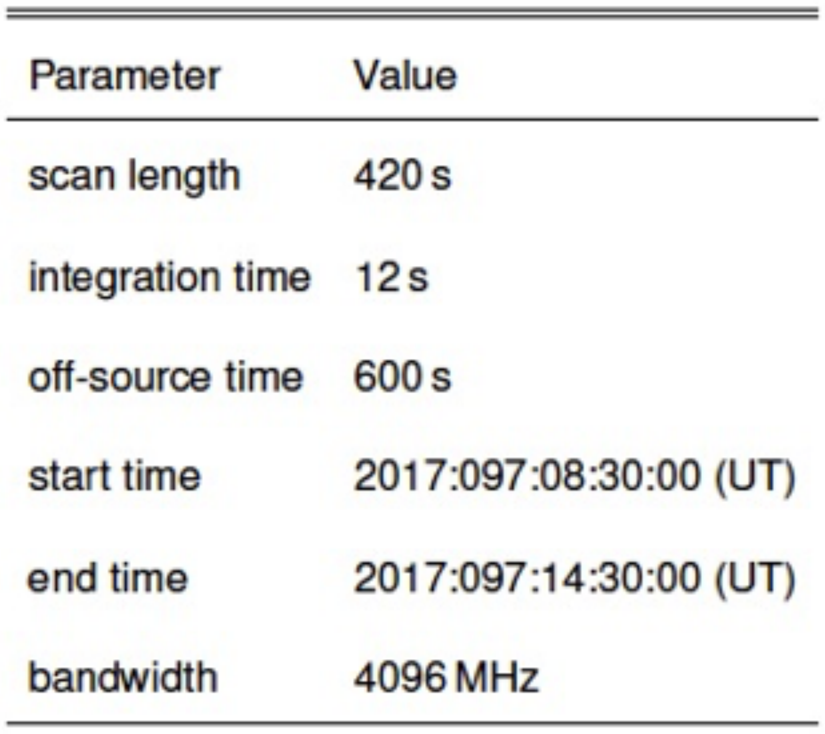
Synthetic Imaging (VLBI array)

- Consider realistic properties of VLBI array & stations adjusting April 2017 EHT observations
- For the synthetic images we use 6h observation time, 420s scan length, 12s integration time, and include interstellar scattering.

UV plane & visibility amplitude



Chosen observation parameter

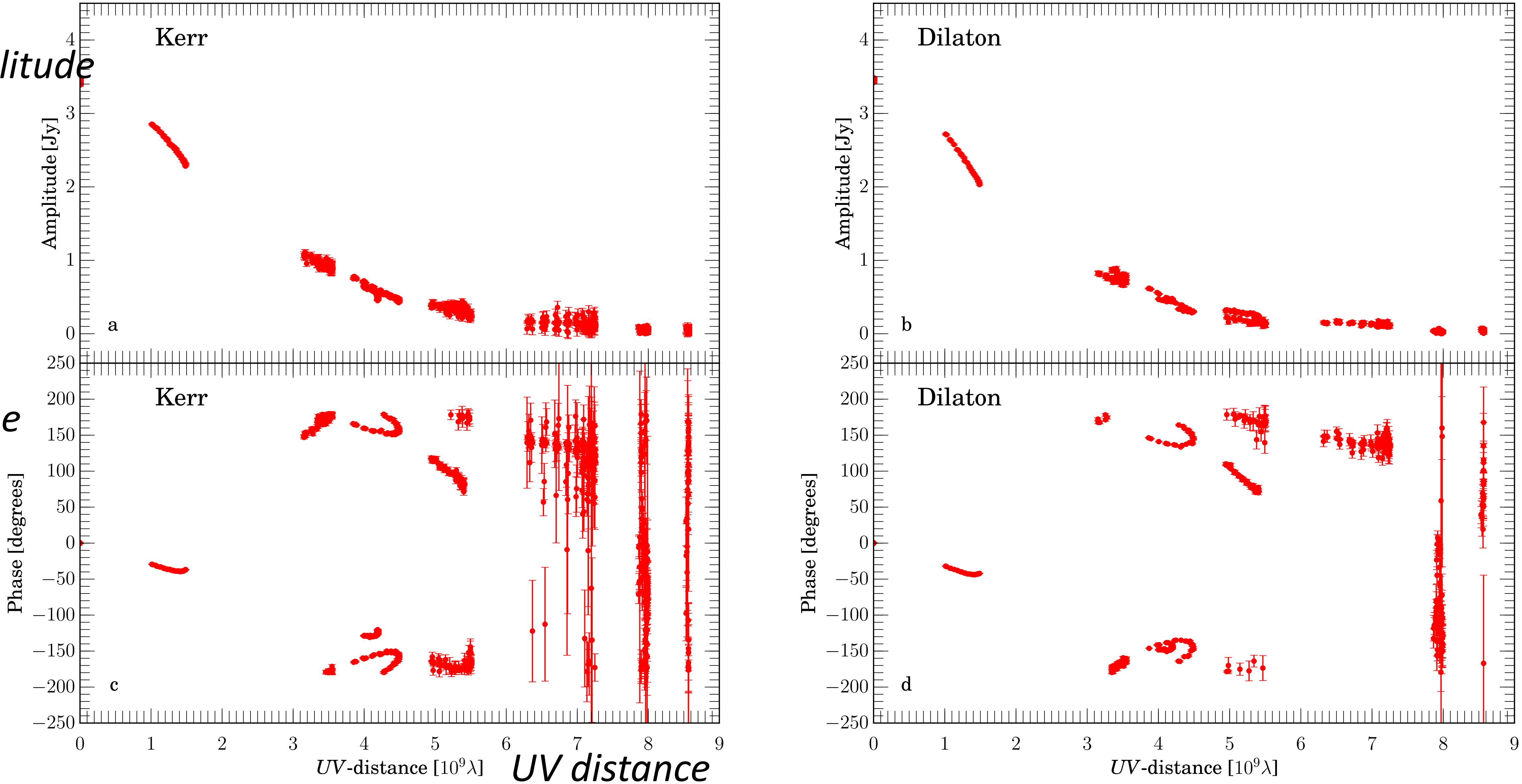


Parameter	Value
scan length	420 s
integration time	12 s
off-source time	600 s
start time	2017:097:08:30:00 (UT)
end time	2017:097:14:30:00 (UT)
bandwidth	4096 MHz

using *ehtim* python modules

Synthetic imaging (visibility amplitude)

Constrained total flux to 3.4Jy in both cases



Amplitude

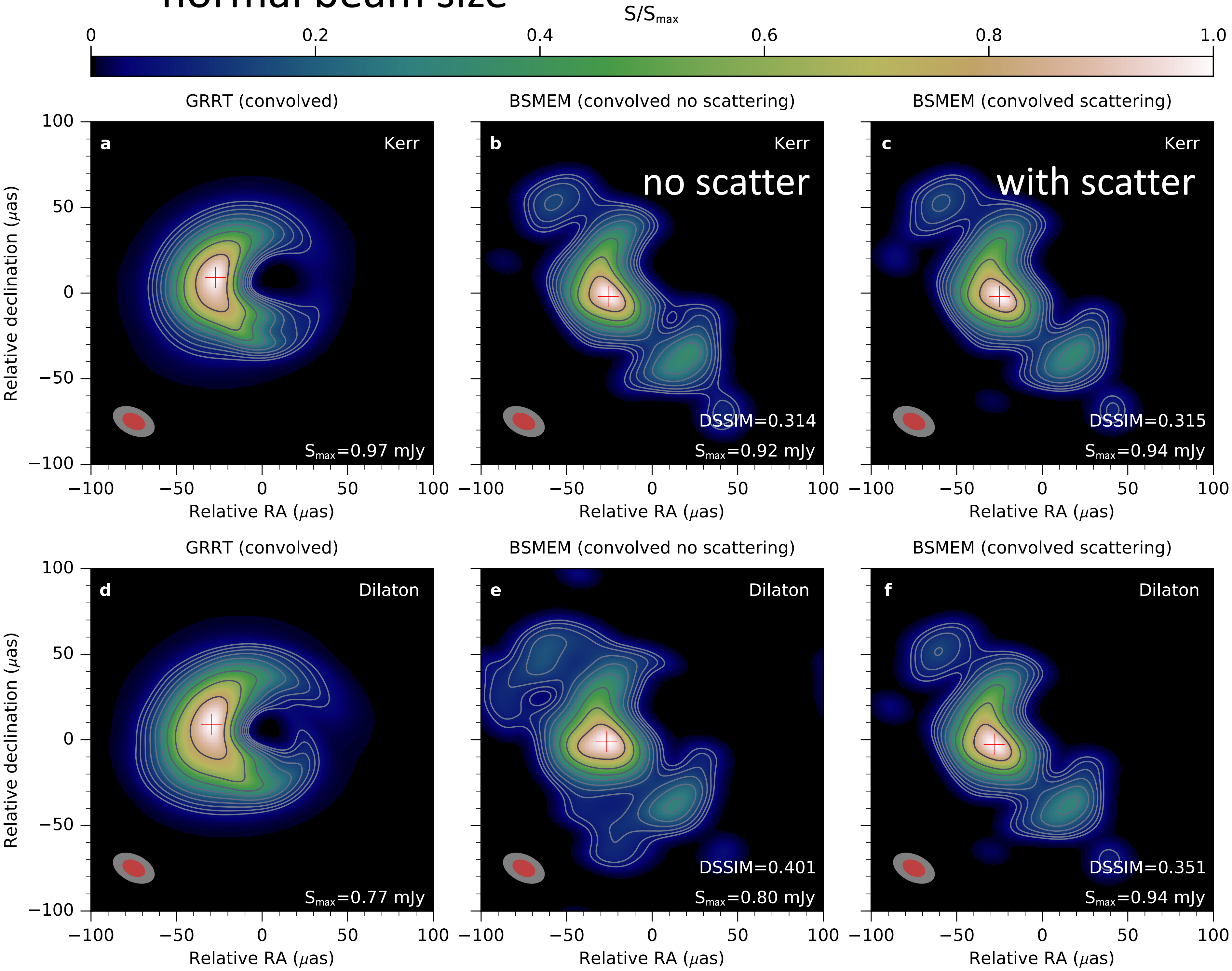
Phase

UV distance

Very similar visibility amplitude and phase in Kerr and dilaton BHs

Synthetic Imaging (shadow image)

reconstruction: BSMEM with 50% normal beam size



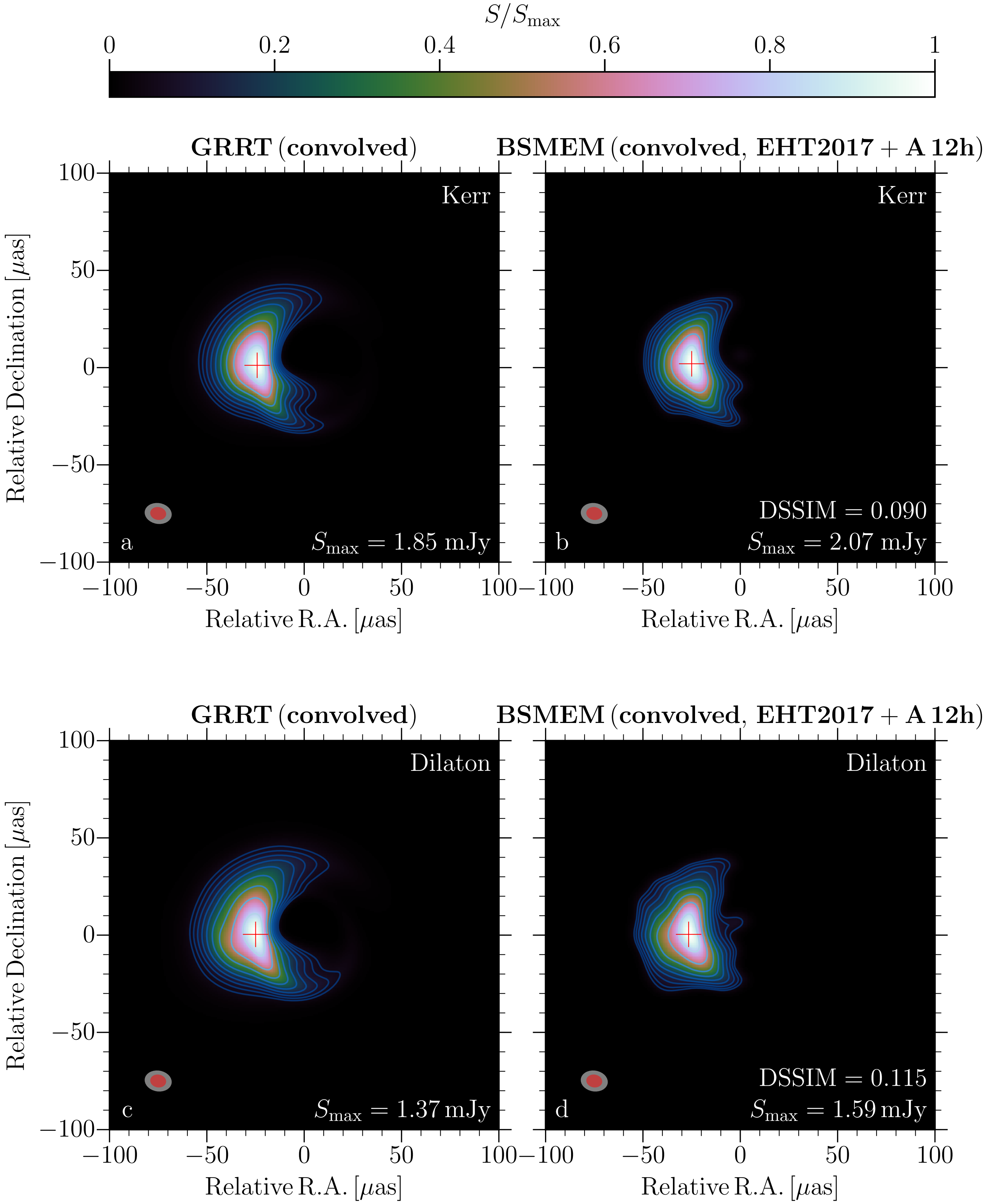
- Convolved GRRT images: already smeared out of sharp emission features
- Reconstructed images: mapped critical features of BH images (e.g., crescent shape)
- interstellar scattering: increases the blurring of these features

Future development: addition two african telescope @ 340 GHz

- Consider addition of two african telescopes + 12 hour observation at 340 GHz with 16GHz bandwidth.

- The reconstructed images agree very well with GRRT convolved images and show very detail features.

- Technological developments will improve the ability to distinguish BH spacetimes from shadow images alone, motivating further work in this direction.



Summary

- There are many application for using GRMHD in the Universe.

- Relativistic jets have still many unsolved problems such as jet formation, acceleration, collimation, magnetic dissipation.

- EHT observations will give us the first BH shadow image and information about jet formation site.

- For realistic theoretical model, coupling with fluid dynamics through GRMHD simulations and radiation calculations (radiation process and radiation transfer) is important.

- Ideal GRMHD equations are the most simple description about the macroscopic plasma in GR regime. The effect of missing physics (radiation, resistivity, non-perfect fluid etc) will be investigated.

- It would be very important for coupling with macroscopic plasma and microscopic plasma process.