Hydrodynamics and Magnetohydrodynamics: Solutions of the exercises in Lecture XV

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Lecture XV, Exercise 1.

The Debye length and plasma parameter are defined respectively as

$$\lambda_D := \left(\frac{k_B T}{4\pi n e^2}\right)^{1/2}, \tag{1}$$

$$\Lambda := 4\pi n \lambda_D^3. \tag{2}$$

Assuming that the number density and temperature of the electrons are the same as those of protons we conclude that the Debye length and plasma parameters are also the same for electrons and protons.

We list the results for the the different physical conditions suggested:

	λ_D [cm]	Λ
fusion machine	2.2×10^{-4}	1.3×10^{6}
Earth's magnetosphere	2.2	1.3×10^{6}
center of the Sun	2.7×10^{-9}	26.0
solar corona	6.9×10^{-1}	4.1×10^{-8}
solar wind	6.9×10^{2}	4.1×10^{10}
neutron star's atmosphere	2.2×10^{-2}	1.3×10^{9}

Note that with the exception of the solar corona, $\Lambda > 1$ and more often $\Lambda \gg 1$.

Lecture XV, Exercise 2.

The collision frequency for an electron in a plasma at temperature T is given by

$$f_{\rm coll} := \frac{\sqrt{2}\omega_{P,e}^4}{64\pi n_e}\left(\frac{k_B T}{m_e}\right)^{-3/2} \ln\Lambda. \tag{3}$$

and we can compare it with the plasma frequency $\omega_{P,e} =: f_{P,e}/(2\pi)$, which we recall is given by

$$\omega_{P,e} = \sqrt{\frac{4\pi n_e e^2}{m_e}}. \tag{4}$$

The result of this comparison is shown in the table below for the various physical suggested:

	f_{coll} [Hz]	$\omega_{P,e}$ [Hz]
fusion machine	7.0×10^{16}	9.0×10^{11}
Earth's magnetosphere	7.0×10^{8}	9.9×10^{5}
center of the Sun	1.3×10^{26}	9.0×10^{16}
solar corona	1.0×10^{10}	9.0×10^{7}
solar wind	1.2×10^{3}	2.8×10^{4}
neutron star's atmosphere	1.4×10^{13}	9.0×10^{9}

Note that when the collision frequency is smaller than the plasma frequency, i.e., $f_{\mathrm{coll}} < f_{P,e}$, the collisions do not effect the plasma oscillations. This is usually referred to as a "weakly coupled plasma" or "collisionless plasma", and is produced in plasmas that are diffused and at high temperatures. When browsing the table above it is clear that only a solar wind can be considered as a collisionless plasma.